

Sequences

Series

Patterns

Text & Tests 6

Chapter 4

Test Solutions

5th Year
HL Maths

Q1
geometric

$$T_n = ar^{n-1}$$

1, $\frac{1}{2}$, $\frac{1}{4}$... $r = ?$ $T_n = ?$
 $\times \frac{1}{2}$ $\times \frac{1}{2}$

$$r = \frac{1}{2} \quad a = 1$$

$$T_n = 1\left(\frac{1}{2}\right)^{n-1}$$

Q2

$$T_n = ar^{n-1}$$

$$T_2 = 21 \quad T_3 = -63 \quad \text{GEOMETRIC}$$

$$\Rightarrow ar = 21 \quad ar^2 = -63$$

$$\frac{T_3}{T_2} = \frac{ar^2}{ar} = r = \frac{-63}{21} = -3$$

$$a(-3) = 21 \Rightarrow a = -7$$

n=4

$$T_n = -7(-3)^{n-1}$$

n=5

$$T_5 = -7(-3)^4 = -567$$

OR

$$-7, 21, -63, 189, -567$$

Q3

Sum?

$$\sum_{r=1}^8 2(3^r) = ?$$

r=1

$$T_1 = 2(3^1) = 6$$

r=2

$$T_2 = 2(3^2) = 18$$

⋮

$$T_3 = 2(3^3) = 54$$

$$T_4 = 162$$

$$T_5 = 486$$

$$T_6 = 1458$$

$$T_7 = 4374$$

$$T_8 = 13122$$

$$\text{Sum} = 19,680$$

Q4

Cubic expression? " $an^3 + bn^2 + cn + d$ "

	T_1	T_2	T_3		
	-1	2	17	50	107
D_1		3	15	33	57
D_2			12	18	24
D_3				6	6

3rd difference = 6 = 6a $\Rightarrow a = 1$

Cubic = $n^3 + bn^2 + cn + d$

$$T_1 = (1)^3 + b(1)^2 + c(1) + d = -1$$

$$1 + b + c + d = -1$$

$$b + c + d = -2 \quad (1)$$

$$T_2 = (2)^3 + b(2)^2 + c(2) + d = 2$$

$$8 + 4b + 2c + d = 2$$

$$4b + 2c + d = -6 \quad (2)$$

$$T_3 = (3)^3 + b(3)^2 + c(3) + d = 17$$

$$27 + 9b + 3c + d = 17$$

$$9b + 3c + d = -10 \quad (3)$$

$$b + c + d = -2 \quad (1)$$

$$4b + 2c + d = -6 \quad (2)$$

$$9b + 3c + d = -10 \quad (3)$$

 $(2) - (1)$

$$4b + 2c + d = -6$$

$$-b - c - d = +2$$

$$3b + c = -4 \quad (4)$$

 $(3) - (1)$

$$9b + 3c + d = -10$$

$$-b - c - d = +2$$

$$8b + 2c = -8 \Rightarrow 4b + c = -4 \quad (5)$$

 $(5) - (4)$

$$4b + c = -4$$

$$-3b - c = +4$$

$$b = 0 \Rightarrow 4(0) + c = -4$$

$$c = -4$$

 $\rightarrow (1)$

$$0 + -4 + d = -2 \Rightarrow d = 2$$

Cubic = $1n^3 + 0n^2 + -4n + 2$

$$n^3 - 4n + 2$$

Q5

ARITHMETIC

$T_3 = 71$

$T_7 = 55$

$a = ? \quad d = ? \quad T_n = ? \quad T_{10}$

$T_n = a + (n-1)d$

$\Rightarrow a + 2d = 71 \quad (1)$

$a + 6d = 55 \quad (2)$

 $(1) - (2)$

$$\begin{array}{r} d + 2d = 71 \\ - a - 6d = -55 \\ \hline -4d = +16 \\ d = -4 \end{array}$$

$$\begin{aligned} \Rightarrow a + 2(-4) &= 71 \\ a - 8 &= 71 \\ a &= 79 \end{aligned}$$

$$T_n = 79 + (n-1)(-4) \quad \checkmark$$

$$= 79 - 4n + 4$$

$T_n = 83 - 4n$

$T_{10} = 83 - 4(10) = 43 = T_{10}$