Example 5

(i) Use the sigma notation ($\sum$) to represent $2 + 6 + 10 + 14 + \ldots\ldots$ for 45 terms.

(ii) For what value of n is $\sum_{r=1}^{n} 3r - 5 = 90$?

(iii) Find the value of $\sum_{r=1}^{8} 4r - 1$.

$\sum$ = "sum"

Sigma notation explained.

The sum of the first 45 terms is usually written as S_{45}.
But this can also be written in "sigma notation".

$$S_{45} = \sum_{r=1}^{r=45} T_r$$

this means "the sum of terms from term 1 $(r=1)$ up to term 45 $(r=45)$

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$T_n = a + (n-1)d$

(i) $a = 2$, $d = 4$

$$S_{45} = \sum_{r=1}^{45} [a + (r-1)d] = \sum_{r=1}^{45} (2 + (r-1)4)$$

$$= \sum_{r=1}^{45} (2 + 4r - 4) = \sum_{r=1}^{45} 4r - 4$$

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(ii)

$\sum_{r=1}^{n}(3r-5) = 90$, $n = ?$

change to "normal notation"

$T_n = 3n - 5 \Rightarrow T_1 = 3(1) - 5 = -2 = a$

$T_2 = 3(2) - 5 = 1$

$S_n = 90$

$d = T_2 - T_1 = 1 - (-2) = 3 \Rightarrow d = 3$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 90 = \frac{n}{2}[2(-2) + (n-1)(3)]$

Solve quadratic

$180 = n(-4 + 3n - 3)$

$180 = n(-7 + 3n)$

$180 = -7n + 3n^2$

$3n^2 - 7n - 180 = 0$

$(3n + 20)(n - 9) = 0$

$n = -20/3$ ✗ or $n = 9$ ✓

Example 5

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(iii)

change to "normal notation"

$T_n = 4n - 1 \Rightarrow T_1 = 4(1) - 1 = 3 = a$

$T_2 = 4(2) - 1 = 7$

$\Rightarrow d = 4$

$n = 8$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_8 = \frac{8}{2}[2(3) + (8-1)(4)]$

$= 4[6 + 28] = 4[34]$

$= 136$

5. Anna saves money each week to buy a printer which costs €190. Her plan is to start with €10 and to put aside €2 more each week (i.e. €12, €14, etc.) until she has enough money to buy the printer.
At this rate, how many weeks will it take Anna to save for the printer?

Series = $10 + 12 + 14 + \ldots$

$a = 10$, $d = 2$, $S_n = 190$, $n = ?$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$190 = \frac{n}{\cancel{2}}[\cancel{2}(10) + (n-1)\cancel{2}]$

$190 = n[10 + n - 1]$

$190 = n[9 + n]$

$190 = 9n + n^2$

Solve quadratic

$\Rightarrow n^2 + 9n - 190 = 0$

$(n + 19)(n - 10) = 0$

$\Rightarrow n = -19$ ✗ or $n = 10$ ✓

−19 weeks makes no sense!

6. Evaluate (i) $\sum_{r=1}^{6}(3r+1)$ (ii) $\sum_{r=0}^{5}(4r-1)$ (iii) $\sum_{r=1}^{100} r$

(i) $S_6 = ?$

$T_1 = 3(1) + 1 = 4 = a$

$T_2 = 3(2) + 1 = 7$

$\Rightarrow d = 3$

$n = 6$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_6 = \frac{6}{2}[2(4) + (6-1)3]$

$= 3[8 + (5)3] = 3[8 + 15]$

$= 3[23] = 69$ ✓

6. Evaluate (i) $\sum_{r=1}^{6}(3r+1)$ (ii) $\sum_{r=0}^{5}(4r-1)$ (iii) $\sum_{r=1}^{100} r$

(ii)

$$\sum_{r=0}^{5}(4r-1)$$

Careful here:
In this example the first term is when $r=0$!
We add terms 'till $r=5$
$\Rightarrow$ that means 6 terms.

$T_1 = 4(0)-1 = -1$
$T_2 = 4(1)-1 = 3$
$\Rightarrow$ $d = 4$

Sum of first 6 terms?

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_6 = \frac{6}{2}[2(-1)+(6-1)4]$
$= 3[-2+(5)4] = 3[-2+20]$
$= 3[18] = 54$ ✓

6. Evaluate (i) $\sum_{r=1}^{6}(3r+1)$ (ii) $\sum_{r=0}^{5}(4r-1)$ (iii) $\sum_{r=1}^{100} r$

$T_1 = 1 = a$
$T_2 = 2$
$\Rightarrow$ $d = 1$
$n = 100$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{100} = \frac{100}{2}[2(1)+(100-1)1]$
$= 50[2+99] = 50[101]$
$= 5050$ ✓

7. Write each of the following series in sigma notation.
(i) $4 + 8 + 12 + 16 + \ldots\ldots 124$ (ii) $-10 - 9\frac{1}{2} - 8 - 7\frac{1}{2} + \ldots\ldots 4$
(iii) $10 + 10.1 + 10.2 + 10.3 + \ldots\ldots 50$

(i)

$a = 4$, $d = 4$, $T_n = 124$, $n = ?$

expression for T_n

$T_n = a + (n-1)d$

$T_n = 4 + (n-1)4$
$= 4 + 4n - 4$
$T_n = 4n$

$n = ?$

$124 = 4n$
$n = 31$

series in sigma notation

$$\sum_{r=1}^{31} 4r \checkmark$$

7. Write each of the following series in sigma notation.
(i) $4 + 8 + 12 + 16 + \ldots\ldots 124$ (ii) $-10 - 9\frac{1}{2} - 8 - 7\frac{1}{2} + \ldots\ldots 4$
(iii) $10 + 10.1 + 10.2 + 10.3 + \ldots\ldots 50$

(ii)

$a = -10$, $d = \frac{1}{2}$, $T_n = 4$, $n = ?$

expression for T_n

$T_n = a + (n-1)d$

$T_n = -10 + (n-1)\frac{1}{2}$
$T_n = -10 + \frac{n}{2} - \frac{1}{2}$

$T_n = -10\frac{1}{2} + \frac{n}{2} \Rightarrow T_n = \frac{n-21}{2}$

$T_n = 4 = \frac{n-21}{2}$

$n = ?$

$\Rightarrow 8 = n - 21$
$29 = n$

series in sigma notation

$$\sum_{r=1}^{29} \frac{r-21}{2} \checkmark$$

7. Write each of the following series in sigma notation.

(i) $4 + 8 + 12 + 16 + \ldots\ldots 124$ (ii) $-10 - 9\frac{1}{2} - 8 - 7\frac{1}{2} + \ldots\ldots 4$

(iii) $10 + 10.1 + 10.2 + 10.3 + \ldots\ldots 50$

(iii)

$a = 10$, $d = 0.1$, $T_n = 50$

expression for T_n

$T_n = a + (n-1)d$

$T_n = 10 + (n-1)0.1$

$T_n = 10 + 0.1n - 0.1$

$T_n = 9.9 + 0.1n$

$T_n = \frac{99 + n}{10}$

$n = ?$

$T_n = \frac{99+n}{10} = 50$

$\Rightarrow 99 + n = 500$

$n = 401$

series in sigma notation

$$\sum_{n=1}^{401} \frac{99+r}{10}$$ ✓

note: mistake in book answer

8. In an arithmetic series, $T_4 = 15$ and $S_5 = 55$.
Find the first five terms of the series.

$T_n = a + (n-1)d$

$T_4 = a + (4-1)d = 15$

$a + 4d - 4 = 15$

$a + 4d = 19$ ①

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_5 = \frac{5}{2}[2a + (5-1)d] = 55$

$\Rightarrow 5[2a + 4d] = 110$

$\Rightarrow 2a + 4d = 22$

$a + 2d = 11$ ②

Solve

① − ② $\Rightarrow 2d = 8 \Rightarrow d = 4$ ③

③ → ② $\Rightarrow a + 2(4) = 11$

$a = 11 - 8 \Rightarrow a = 3$

$\Rightarrow$ First 5 terms

$3 + 7 + 11 + 15 + 19 \ldots$

9. The third term of an arithmetic sequence is 18 and the seventh term is 30. Find the sum of the first 33 terms.

$T_n = a + (n-1)d$

$T_3 = a + (3-1)d = 18$

$a + 2d = 18$ ①

Solve

$T_7 = a + (7-1)d = 30$

$a + 6d = 30$ ②

① - ② $\Rightarrow 4d = 12 \Rightarrow d = 3$ ③

③ → ② $\Rightarrow a + 6(3) = 30$

$a = 30 - 18 \Rightarrow a = 12$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{33} = \frac{33}{2}[2(12) + (33-1)3]$

$= \frac{33}{2}[24 + 96] = \frac{33}{2}[120]$

$= 1980$ ✓