

3. How many terms of the series  $5 + 8 + 11 + 14 + \dots$  must be added to make a total of 98?

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$a = 5$$

$$d = 3$$

$$S_n = 98$$

$$98 = \frac{n}{2} [2(5) + (n-1)3]$$

$$2(98) = n [10 + 3n - 3]$$

$$196 = n [7 + 3n]$$

$$196 = 7n + 3n^2$$

$$3n^2 + 7n - 196 = 0$$

$$(3n + 28)(n - 7) = 0$$

$$\begin{array}{l|l} 3n + 28 = 0 & n = 7 \\ n = -\frac{28}{3} & \end{array}$$

4. Given  $T_n = 5 - 3n$ , write down the first term  $a$ , and the common difference  $d$ . Hence find  $S_{10}$ .

$$a = 2$$

$$d = -3$$

$$T_1 = 5 - 3(1) = 2$$

$$T_2 = 5 - 3(2) = -1$$

$$d = T_2 - T_1 = -1 - 2 = -3$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{10} = \frac{10}{2} [2(2) + (10-1)(-3)]$$

$$= 5 [4 - 27]$$

$$= 5 [-23]$$

$$= -115$$

**Example 1**

Find the sum of the series  $4 + 11 + 18 + 25 + \dots + 144$ .

$$S_1 = T_1 = 4$$

$$S_2 = 15$$

$$T_2 = 15 - 4 = 11 \quad \checkmark$$

$$T_2 = S_2 - T_1$$

**Example 2**

To celebrate the birth of his niece, an uncle offers to open a savings account with a deposit of €50. He also offers to every year add €10 more than he did the previous year until his niece is 21 years of age.

- (i) Find an expression for  $S_n$ , the sum of money on deposit after  $n$  years.  
 (ii) Find  $S_{21}$ , the total saved after 21 years.

$$\begin{array}{ccccccc}
 \textcircled{0} & & 1 & & & & \textcircled{21} \\
 50 & + & 60 & \dots & \dots & \dots & \\
 n=1 & & n=2 & & & & n=22
 \end{array}$$

**Example 3**

Given  $S_n = n^2 - 4n$ , find an expression for  $T_n$  and hence determine if the sequence is arithmetic.

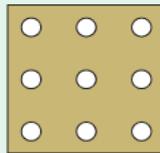
$$\begin{aligned}
 T_1 &= S_1 = (1)^2 - 4(1) = 1 - 4 = -3 \\
 S_2 &= (2)^2 - 4(2) = 4 - 8 = -4 \\
 T_2 &= S_2 - T_1 = -4 - (-3) = -1 \\
 &\quad \overbrace{-3, -1, 1, 3, \dots} \\
 a &= -3 \\
 d &= 2 \\
 T_n &= a + (n-1)d \\
 T_n &= -3 + (n-1)2 \\
 &= -3 + 2n - 2 = -5 + 2n \\
 \text{check } T_3 &= -5 + 2(3) = -5 + 6 = 1 \checkmark
 \end{aligned}$$

**Example 4**

A lighting company is making a sequence of light panels with the number of bulbs per panel in arithmetic sequence.

For the first 10 panels, 165 bulbs were used.

If the third panel is as shown in the diagram, find  $a$ , the first term of the sequence, and  $d$ , the common difference.



3rd panel (9 bulbs)

Hence draw a diagram of the first four panels.

**Example 5**

(i) Use the sigma notation ( $\sum$ ) to represent  $2 + 6 + 10 + 14 + \dots$  for 45 terms.

(ii) For what value of  $n$  is  $\sum_{r=1}^n 3r - 5 = 90$  ?

(iii) Find the value of  $\sum_{r=1}^8 4r - 1$ .

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