

**Example 5**

Find the number of terms in the geometric sequence  $81, 27, 9, \dots, \frac{1}{27}$ .

$n = ?$

$T_n = ar^{n-1}$

$3^3 = 27$

$3^4 = 81$

$(3^3)(3^4) = 3^7$

$a = 81$        $r = \frac{27}{81} = \frac{1}{3}$        $T_n = \frac{1}{27}$

$\Rightarrow \frac{1}{27} = 81 \left(\frac{1}{3}\right)^{n-1}$

$\Rightarrow \frac{1}{(27)(81)} = \left(\frac{1}{3}\right)^{n-1}$

$\frac{1}{3^7} = \frac{1}{3^{n-1}}$

$7 = n - 1$   
 $n = 8$  ✓

alternative method

$\frac{1}{2187} = \frac{1}{3^{n-1}}$

$3^? = 2187$

$\log_3 2187 = 7$

$\Rightarrow n = 8$

Sum to n terms of geometric series

eg. (i)

$T_5$  and  $S_5 = ?$

(i)  $1 + 3 + 9 + \dots$        $a = 1, r = 3$

$T_n = ar^{n-1}$

$T_5 = (1)(3)^{5-1} = 3^4 = 81$  ✓

$S_n = \frac{a(1-r^n)}{1-r}$

$S_5 = \frac{1(1-3^5)}{1-3} = \frac{1-3^5}{-2} = \frac{1-243}{-2} = 121$  ✓

example 2

$$T_n = ar^{n-1}$$

Solve

$$T_3 = 32 \quad T_6 = 4$$

Find  $a$ ,  $r$  &  $S_8$ ?

$$32 = ar^2 \quad \text{and} \quad 4 = ar^5$$

$$a = 32/r^2 \quad \Rightarrow \quad 4 = \left(\frac{32}{r^2}\right)r^5$$

$$\Rightarrow 4 = 32r^3$$

$$\Rightarrow \left(\frac{4}{32}\right) = r^3$$

$$\Rightarrow \sqrt[3]{\frac{1}{8}} = r = \frac{1}{2} \quad \checkmark$$

$$\Rightarrow a = 32 / \left(\frac{1}{2}\right)^2 = 32(4) = 128 = a \quad \checkmark$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_8 = \frac{128(1 - (\frac{1}{2})^8)}{1 - \frac{1}{2}} = 255 \quad \checkmark$$

P.160  
Ex. 4.5Q4 Series:  $32 + 16 + 8 + \dots$   $S_{10} = ?$ 

$$a = 32 \quad r = \frac{16}{32} = \frac{1}{2}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{10} = \frac{32(1 - (\frac{1}{2})^{10})}{1 - \frac{1}{2}}$$

$$\approx 63.94$$

Q5

Series:  $4 - 12 + 36 - 108 + \dots$

$S_6 = ?$

$$a = 4 \quad r = \frac{-12}{4} = -3$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_6 = \frac{4(1-(-3)^6)}{1-(-3)}$$

$$= -728$$

Q6

Series:  $729 - 243 + 81 - \dots - \frac{1}{3}$

Find the number of terms?

$n = ?$

$$a = 729, \quad r = \frac{-243}{729} = -\frac{1}{3}, \quad T_n = -\frac{1}{3}$$

$$T_n = ar^{n-1}$$

$$\Rightarrow -\frac{1}{3} = 729 \left(-\frac{1}{3}\right)^{n-1}$$

divide by 729

$$\frac{1}{(-3)(729)} = \frac{1}{(-3)^{n-1}}$$

$$(3)(729) = 2187$$

$$\log_3 2187 = 7$$

$$\Rightarrow \frac{1}{(-3)^7} = \frac{1}{(-3)^{n-1}} \quad \Rightarrow n-1 = 7$$

$$\Rightarrow n = 8$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_8 = \frac{729(1-(-\frac{1}{3})^8)}{1-(-\frac{1}{3})} = 546\frac{2}{3}$$

HW ex 4.5 Q.4-7

Q7

$$\sum_{r=1}^6 4^r$$

Sum terms  
from input  
 $r=1$  to  $r=6$

 $T_n$ 

$$T_1 = 4^1 = 4$$

$$T_2 = 4^2 = 16$$

$$T_3 = 4^3 = 64$$

$$a = 4$$

$$r = 4$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_6 = \frac{4(1-4^6)}{1-4} = 5460$$