(1) Differentiate $\sqrt{x}(x+2)$ with respect to x [10 MARKS]

MULTIPLYING METHOD

$$y = \sqrt{x}(x+2) = x^{\frac{1}{2}}(x+2) = x^{3/2} + 2x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + x^{-\frac{1}{2}}$$

PRODUCT METHOD

$u = \sqrt{x}$ $\quad v = (x+2)$

$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ $\quad \frac{dv}{dx} = 1$

$$\frac{dy}{dx} = (\sqrt{x})(1) + (x+2)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$= x^{\frac{1}{2}} + \frac{1}{2}x^{\frac{1}{2}} + x^{-\frac{1}{2}}$$

$$= \frac{3}{2}x^{\frac{1}{2}} + x^{-\frac{1}{2}}$$

(2) Differentiate $\cos^2 x$ with respect to x. [10 MARKS]

$$f(x) = (\cos x)^2$$

$$f'(x) = 2(\cos x)^1(-\sin x)$$

$$= -2\cos x \sin x$$

3 The equation of a curve is $y = e^{-x^2}$. [10 MARKS]

(i) Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = e^{-x^2}(-2x) = -2x e^{-x^2}$$

4 Find the slope of the tangent to the curve $x^2 + y^3 = x - 2$ at the point $(3, -2)$.

[10 MARKS]

$$2x + 3y^2 \frac{dy}{dx} = 1$$

$$3y^2 \frac{dy}{dx} = 1 - 2x$$

$$\frac{dy}{dx} = \frac{1-2x}{3y^2} \quad \text{at } (3,-2) \quad = \frac{1-2(3)}{3(-2)^2} = \frac{-5}{12}$$

(5) Differentiate with respect to x:

(i) $(4x^2-1)^3$.

(ii) $\sin^{-1}\left(\frac{2x}{3}\right)$. [20 MARKS]

(i) $f(x) = (4x^2-1)^3$

$f'(x) = 3(4x^2-1)^2(8x)$

$= 24x(4x^2-1)^2$

(ii) $f(x) = \sin^{-1}\left(\frac{2x}{3}\right)$

$$\boxed{y = \sin^{-1}\left(\frac{x}{a}\right) \qquad \frac{dy}{dx} = \frac{1}{\sqrt{a^2-x^2}}}$$

$\frac{dy}{dx} = \frac{1}{\sqrt{9-4x^2}}(2)$

$= \frac{2}{\sqrt{9-4x^2}}$

(6) A curve is defined by the equation $x^2 - 2xy + 3y^2 + 4y = 22$.

(i) Find $\frac{dy}{dx}$ in terms of x and y. [10 MARKS]

$2x - 2x\frac{dy}{dx} - 2y + 6y\frac{dy}{dx} + 4\frac{dy}{dx} = 0$

$\frac{dy}{dx}(-2x+6y+4) = 2y - 2x$

$\frac{dy}{dx} = \frac{\cancel{2}(y-x)}{+\cancel{2}(-x+3y+2)} = \frac{x-y}{x-3y-2}$

ASIDE

$2xy$ = PRODUCT

$u = 2x \qquad v = y$

$\frac{du}{dx} = 2 \qquad \frac{dv}{dx} = \frac{dy}{dx}$

DERIVATIVE

$2x\frac{dy}{dx} + 2y$

(7) $x = e^t \cos t$ and $y = e^t \sin t$. Show that $\dfrac{dy}{dx} = \dfrac{x+y}{x-y}$. [10 MARKS]

$$\frac{dx}{dt} = e^t(-\sin t) + e^t \cos t = e^t \cos t - e^t \sin t$$

$$\frac{dy}{dt} = e^t(\cos t) + e^t \sin t$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{e^t \cos t + e^t \sin t}{e^t \cos t - e^t \sin t} = \frac{x+y}{x-y}$$

(8) A curve is defined by the parametric equations [20 MARKS]

$x = \dfrac{t-1}{t+1}$ and $y = \dfrac{-4t}{(t+1)^2}$, where $t \neq -1$.

(i) Find $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$.

(ii) Hence find $\dfrac{dy}{dx}$, and express your answer in terms of x.

QUOTIENT

$$\frac{dx}{dt} = \frac{(t+1)(1) - (t-1)(1)}{(t+1)^2}$$

$$= \frac{t+1 - t+1}{(t+1)^2} = \frac{2}{(t+1)^2}$$

QUOTIENT

$$\frac{dy}{dt} = \frac{(t+1)^2(-4) - (-4t)(2(t+1)(1))}{(t+1)^4}$$

$$= \frac{-4(t^2+2t+1) + 4t(2t+2)}{(t+1)^4} = \frac{-4t^2 - 8t - 4 + 8t^2 + 8t}{(t+1)^4}$$

$$= \frac{4t^2-4}{(t+1)^4} = \frac{4(t^2-1)}{(t+1)^4} = \frac{4(t+1)(t-1)}{(t+1)^4} = \frac{4(t-1)}{(t+1)^3}$$

(ii) $$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{4(t-1)}{(t+1)^3} \cdot \frac{(t+1)^2}{2} = \frac{2(t-1)}{(t+1)} = 2x$$