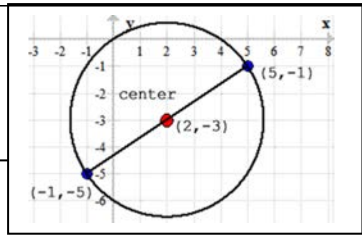
Co-Ordinate Geometry – The Circle



Equation of a circle with centre $(\mathbf{0}, \mathbf{0})$ and radius $\boldsymbol{r}$

$x^2 + y^2 = r^2$

$x^2 + y^2 = 49$ centre (0,0) and radius 7

Equation of circle with centre $(\boldsymbol{h}, \boldsymbol{k})$ and radius $\boldsymbol{r}$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 4)^2 + (y + 3)^2 = 20$ centre $(4, -3)$ and radius $\sqrt{20}$

General Equation of a circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

Make sure every term is on the left hand side and that the coefficients of x and y are equal to 1

Centre is $(-g, -f)$ which is $-\frac{1}{2}$ the coefficient of x and $-\frac{1}{2}$ the coefficient of y

Radius is $\sqrt{g^2 + f^2 - c}$, providing $g^2 + f^2 - c > 0$

$x^2 + y^2 + 10x - 6y - 2 = 0$

centre $(-5,3)$ and radius $\sqrt{(-5)^2 + (3)^2 - (-2)} = 6$

Circles touch Externally

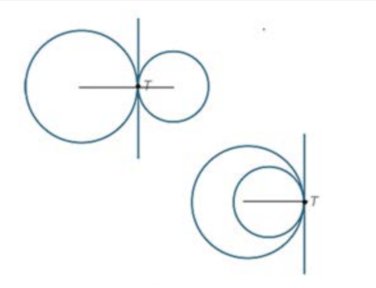
$d = r_1 + r_2$

If circles touch externally the sum of their radii will equal the distance between their centres.

Circles touch Internally

$d = r_1 - r_2$

If circles touch internally the radii of the large circle minus the smaller will equal the distance between their centres.



Using Knowledge from the Line

Distance $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Can help us find the radius given end points of diameter

Midpoint $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Can help us find the midpoint given end points of diameter.

Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation of a line $y - y_1 = m(x - x_1)$

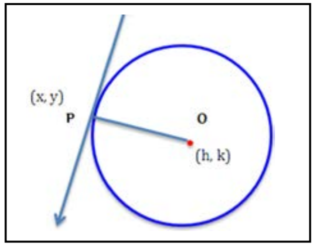
Can help us find equations of tangents to circle.

We can also apply the rules of translations to move circles or find end points of diameter given an endpoint and the centre.

Perpendicular Distance from a Point to a Line

$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

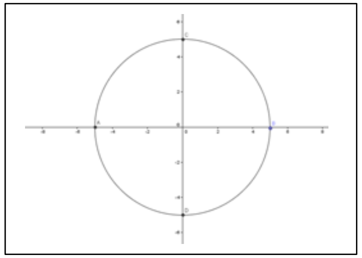
Can be used to find distance from centre of circle to a tangent.



Inside, On or Outside a Circle

Put points into the circle for x and y

$LHS < RHS$	Inside
$LHS = RHS$	On the Circle
$LHS > RHS$	Outside



We can find where a circle:

crosses the x axis by letting $y = 0$

crosses the y axis by letting $x = 0$

Intersection of a Line and a Circle

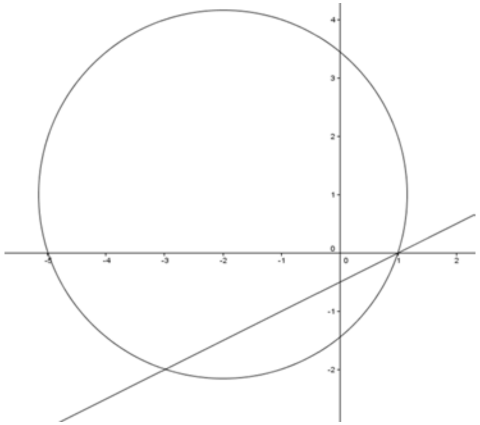
To find the points where a line meets a circle perform a simultaneous equation subbing the x or y value of the line into the circle and solving.

The line $x - 2y - 1 = 0$ *intersects the circle* $x^2 + y^2 + 4x - 2y - 5 = 0$ *at the points* p *and* q. *Find the coordinates of* p *and* q

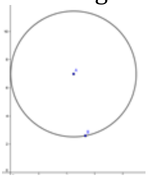
$x = 2y + 1$

$(2y+1)^2 + y^2 + 4(2y+1) - 2y - 5 = 0$
$5y^2 + 10y = 0$
$y^2 + 2y = 0$
$y(y+2) = 0$
$y = 0 \qquad y = -2$
$x = 1 \qquad x = -3$

Thus the coordinates are $(1,0)$ and $(-3,-2)$



Circles with Axes as Tangents
Touching the $x - axis$ $\quad g^2 = c$
Touching the $y - axis$ $\quad f^2 = c$



Proof ($y - axis$)

Radius $= -f$
$\sqrt{g^2 + f^2 - c} = -f$
$g^2 + f^2 - c = f^2$
$g^2 - c = 0$
$g^2 = c$

Finding Equation of Circle with g, f and c

If given 3 points on the circle we can sub in for x and y into:

$x^2 + y^2 + 2gx + 2fy + c = 0$
This gives us three equations in three unknowns g, f and c which we solve and put back into original equation.

Points $p(-4,2)$ $q(-2,6)$ *and* $r(4,8)$ *are on the circle* S. *Find the equation of* S.

Given two points p and q on the circle and the equation of the line L_1, containing the centre $c(-g,-f)$

Sub each point into equation as above to give 2 of the equations.
Sub $(-g,-f)$ in for x and y into line containing the centre to give 3rd equation. Solve all three for g, f and c and sub back into original.

Find the equation of the circle that with points $(-4,1)$ *and* $(0,3)$ *and whose centre lies on the line* $x - 2y + 11 = 0$

Given two points p and q on the circle and the equation of the tangent at one of these points

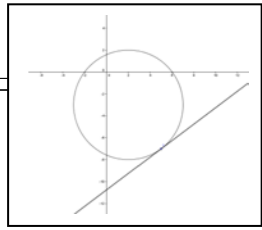
Find the equation of L, the line perpendicular to the tangent T passing through the given point of contact. This point will contain the centre c.
Now we have two points on the circumference of the circle and the equation of the line that contains the centre of the circle. Solve as above.

Find the equation of the circle which passes through the points $a(-3,-2)$ *and* $b(0,-1)$ *and where the line* $2x - y + 4 = 0$ *is a tangent at the point* $a(-3,-2)$

Given the Length of the Radius

When given the radius we let it $\sqrt{g^2 + f^2 - c}$. Then we square both sides. We have to use the other information in the question to form two other equations in g, f and c and sub these into the first equation to get a quadratic equation in one variable. In general we end up with two circles that satisfy the given condition.

A circle of radius length $\sqrt{20}$ *contains the point* $(-1,3)$. *Its centre lies on the line* $x + y = 0$
Find the equations of the two circles that satisfy these conditions.



Equation of a tangent at a given point on the circle

1. Find the slope of the radius to the point of tangency
2. Turn this slope upside down and change the sign. This gives us the slope of the tangent.
3. Use the coordinates of the point of contact and the slope of the tangent at this point in the formula

$$y - y_1 = m(x - x_1)$$

Find the equation of a tangent to the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ *at the point* $(5,-7)$ *on the circle.*