

Coordinate Geometry: The Line

chapter

1

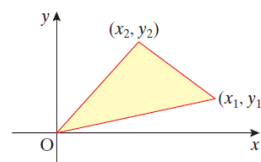
Section 1.2 The area of a triangle

PROJECT MATHS – STRAND 2
Text & Tests 4
LEAVING CERTIFICATE
HIGHER LEVEL

11

The area of the triangle with vertices $(0, 0)$, (x_1, y_1) and (x_2, y_2) is

$$\text{Area} = \frac{1}{2} |x_1 y_2 - x_2 y_1|$$



Example 1

Find the area of the triangle with vertices $(0, 0)$, $(-2, 1)$ and $(3, 4)$.

$$\Delta = \frac{1}{2} |x_1 y_2 - x_2 y_1|$$

$$\begin{aligned} \Delta &= \frac{1}{2} |(-2)(4) - (3)(1)| \\ &= \frac{1}{2} |-8 - 3| \\ &= \frac{1}{2} |-11| \\ &= 11/2 \quad \text{or} \quad 5\frac{1}{2} \text{ units}^2 \end{aligned}$$

Example 2

Find the area of the triangle with vertices $(1, 5)$, $(-3, 1)$ and $(3, -5)$.

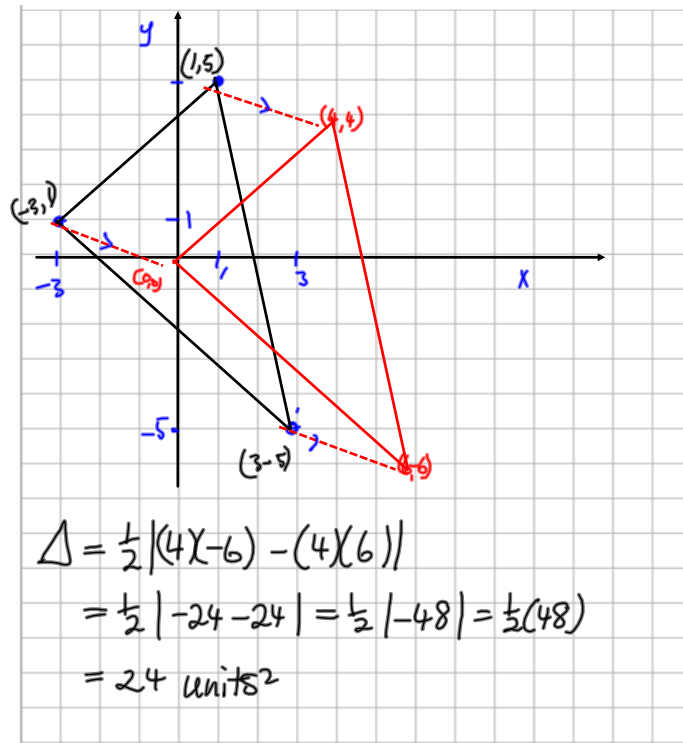
transform one point to the origin

$$(-3, 1) \xrightarrow{+3, -1} (0, 0)$$

$$(1, 5) \longrightarrow (4, 4)$$

$$(3, -5) \longrightarrow (6, -6)$$

$$\Delta = \frac{1}{2} |x_1 y_2 - x_2 y_1|$$



HW Q13 p.10 1.1

1.2 Q3, Q7

notes; absolute value

3. By translating one of the vertices to $(0, 0)$, find the area of each of the triangles whose vertices are

(i) $(2, 3)$, $(5, 1)$ and $(2, 0)$

(ii) $(-2, 3)$, $(4, 0)$ and $(1, -4)$

Step 1 transform
triangle so a vertex
is $(0, 0)$

Step 2 area formula

$$\Delta = \frac{1}{2} |x_1 y_2 + x_2 y_1|$$

$$(2, 0) \xrightarrow{-2, 0} (0, 0)$$

$$(5, 1) \rightarrow (3, 1)$$

$$(2, 3) \rightarrow (0, 3)$$

use
these
points

$$\begin{aligned} \Delta &= \frac{1}{2} |(3)(3) - (1)(0)| = \frac{1}{2} |9 - 0| \\ &= \frac{9}{2} \text{ or } 4.5 \text{ units}^2 \end{aligned}$$

3. By translating one of the vertices to $(0, 0)$, find the area of each of the triangles whose vertices are

(i) $(2, 3)$, $(5, 1)$ and $(2, 0)$

(ii) $(-2, 3)$, $(4, 0)$ and $(1, -4)$

Step 1 transform
triangle so a vertex
is $(0, 0)$

Step 2 area formula

$$\Delta = \frac{1}{2} |x_1 y_2 + x_2 y_1|$$

$$(4, 0) \xrightarrow{-4, 0} (0, 0)$$

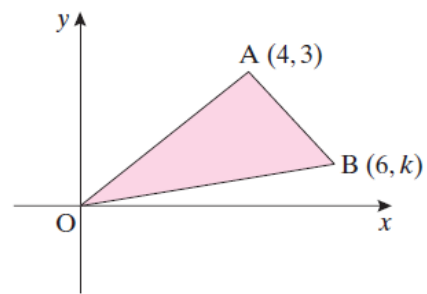
$$(-2, 3) \rightarrow (-6, 3)$$

$$(1, -4) \rightarrow (-3, -4)$$

use these
points

$$\begin{aligned} \Delta &= \frac{1}{2} |(-6)(-4) - (-3)(3)| \\ &= \frac{1}{2} |24 + 9| \\ &= \frac{1}{2} (33) = 16.5 \text{ units}^2 \end{aligned}$$

7. Find the values of k if the area of the given triangle is 7 square units.



$$\Delta = \frac{1}{2} |x_1 y_2 - x_2 y_1|$$

$$7 = \frac{1}{2} |(4)(k) - (6)(3)|$$

$$7 = \frac{1}{2} |4k - 18|$$

$$14 = |4k - 18|$$

2 options : either $4k - 18 = 14$
 $\Rightarrow 4k = 14 + 18$
 $4k = 32$
 $k = 8$ X

or $4k - 18 = -14$
 $4k = -14 + 18$
 $4k = 4$
 $k = 1$ ✓

from diagram we see $k=1$ is correct