

Coordinate Geometry: The Line

chapter

1

Section 1.3 The equation of a line

PROJECT MATHS – STRAND 2
Text & Tests 4
LEAVING CERTIFICATE
HIGHER LEVEL

13

The equation of a line

The equation of a line is generally given in the form $ax + by + c = 0$, e.g., $2x + 3y - 12 = 0$. This may be described as the **general form of the equation of a line**.

To find the equation of a line, we generally need to know

- (i) a **point** on the line
- (ii) the **slope** of the line.

If (x_1, y_1) is a point on the line and m is the slope, we then use $y - y_1 = m(x - x_1)$ to find the equation of the line.

The equation of a line with slope m and containing the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

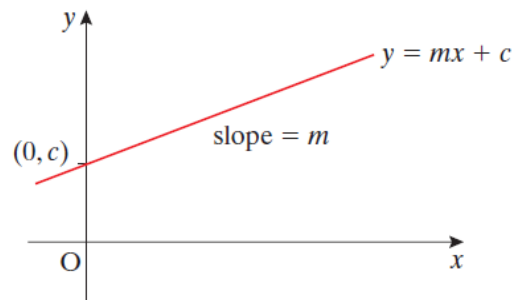
The equation $y = mx + c$

If the equation of a line is in the form

$$y = mx + c, \text{ then}$$

- (i) the slope is m
- (ii) the line intersects the y -axis at $(0, c)$.

The point $(0, c)$ is called the **y-intercept**.



The equation $y = mx + c$ is generally referred to as the **slope-intercept** form of the equation of a line.

Example 1

Find the equation of the line through the point $(\underline{-2}, \underline{3})$ which is perpendicular to the line $2x - y + 5 = 0$.

$$ax + by + c = 0$$

$$m = \frac{-a}{b}$$

equation:

$$y - y_1 = m(x - x_1)$$

slope?

$$m = \frac{-2}{-1} = 2$$

perpendicular?

$$\Rightarrow \perp -\frac{1}{2}$$

$$y - 3 = -\frac{1}{2}(x + 2)$$

$$2(y - 3) = -1(x + 2)$$

$$2y - 6 = -x - 2$$

$$x + 2y - 4 = 0$$

Example 2

Find the value of k if the lines $2x + ky + 5 = 0$ and $(k + 6)x + 2y - 9 = 0$ are perpendicular to each other.

$$m = \frac{-a}{b}$$

$$m_1 \times m_2 = -1$$

$\times 2k$

$$\text{line 1 : } m_1 = \frac{-2}{k}$$

$$\text{line 2 : } m_2 = -\frac{(k+6)}{2}$$

$$\frac{-2}{k} \times \frac{-k-6}{2} = -1$$

$$-2(-k-6) = -1(2)(k)$$

$$+2k + 12 = -2k$$

$$4k = -12$$

$$k = -3$$

3. l is the line $x - 3y + 4 = 0$

- Write down the slope of l .
- Find the equation of the line through $(3, -4)$ parallel to l .

$$m = \frac{-a}{b}$$

parallel slopes
are equal

equation

$$y - y_1 = m(x - x_1)$$

$$m_l = \frac{-1}{-3} = \frac{1}{3}$$

$$y - -4 = \frac{1}{3}(x - 3)$$

$$3(y + 4) = 1(x - 3)$$

$$3y + 12 = x - 3$$

$$x - 3y - 15 = 0$$