

# Geometry 1

chapter

3

## Section 3.3 Ratio theorems

PROJECT MATHS – STRAND 2  
**Text & Tests 4**  
 LEAVING CERTIFICATE  
 HIGHER LEVEL

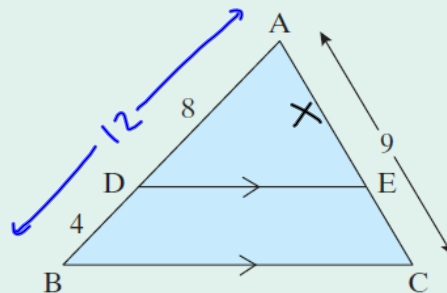
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### Theorem 12

A line drawn parallel to one side of a triangle divides the other two sides in the same ratio.

### Example 1

In the given triangle,  $DE \parallel BC$ .  
 $|AD| = 8$ ,  $|DB| = 4$  and  $|AC| = 9$ .  
 Find  $|AE|$ .



$$\frac{x}{9-x} = \frac{8}{4} = 2$$

$$\begin{aligned} x &= 2(9-x) \\ x &= 18 - 2x \\ 3x &= 18 \\ x &= 6 \end{aligned}$$

*alternate*

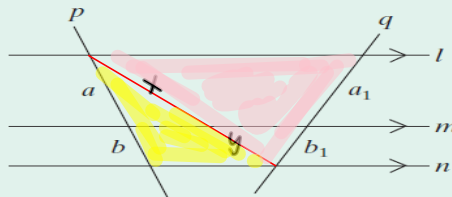
$$\frac{x}{9} = \frac{8}{12}$$

$$x = \frac{2}{3}(9) = 6$$

**Theorem 11**  
 If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.

**Example 2**

In the given figure, the lines  $l, m$  and  $n$  are parallel.



These three lines divide the transversal  $p$  in the ratio  $a : b$ .

The three lines divide the transversal  $q$  in the ratio  $a_1 : b_1$ .

Prove that  $\frac{a}{a_1} = \frac{b}{b_1}$ .

$$\frac{a}{b} = \frac{x}{y}$$

$$\frac{a_1}{b_1} = \frac{x}{y}$$

$$\Rightarrow \frac{a}{b} = \frac{a_1}{b_1}$$

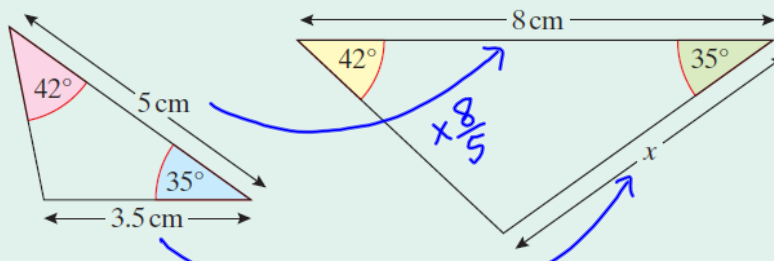
$$\Rightarrow \frac{a}{a_1} = \frac{b}{b_1}$$

**Theorem 13**  
 If two triangles ABC and DEF are similar, then their sides are proportional, in order

$$\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$$

**Example 3**

Find the length of the side marked  $x$  in the triangle below.

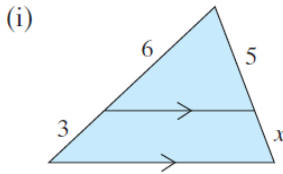


$$3.5 \left(\frac{8}{5}\right) = 5.6$$

scale factor  $k = \frac{8}{5}$

**Exercise 3.3**

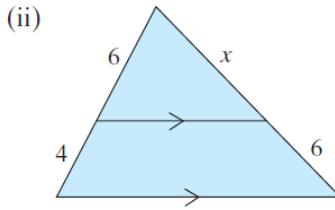
1. In each of the following triangles, the arrows indicate that the lines are parallel. Find the length of the line segment marked  $x$  in each triangle:



$$\frac{x}{5} = \frac{3}{6}$$

$$x = \frac{3(5)}{6}$$

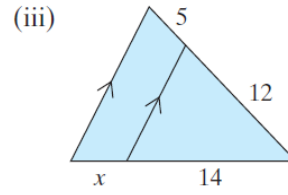
$$x = \frac{5}{2}$$



$$\frac{x}{6} = \frac{6}{4}$$

$$x = \frac{6(6)}{4}$$

$$x = 9$$



$$\frac{x}{14} = \frac{5}{12}$$

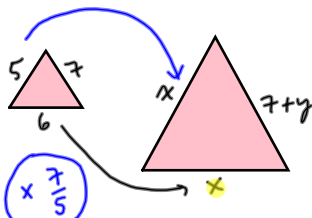
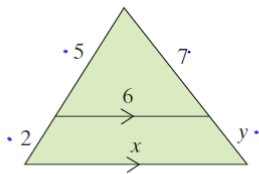
$$x = \frac{5(14)}{12}$$

$$x = \frac{35}{6}$$

**Theorem 12**

2. In the following triangles, the arrows indicate that the lines are parallel. Find the length of the line segment marked with a letter in each triangle:

**Theorem 13**



multiplier

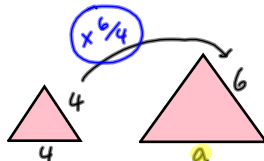
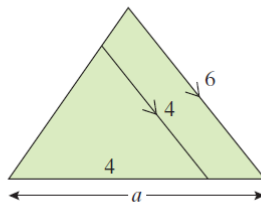
$$x = 6\left(\frac{7}{5}\right) = 8.4$$

$$y = ? \quad 7 + y = 7\left(\frac{7}{5}\right)$$

$$y = \frac{49}{5} - 7$$

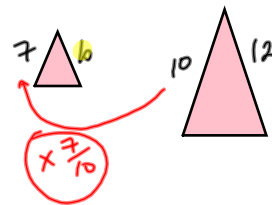
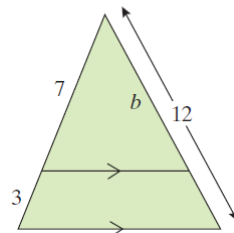
$$y = 2.8$$

scale factor  $k = 7/5$



$$a = 4\left(\frac{6}{4}\right) = 6$$

scale factor  $k = 6/4$



$$b = 12\left(\frac{7}{10}\right)$$

$$b = 8.4$$

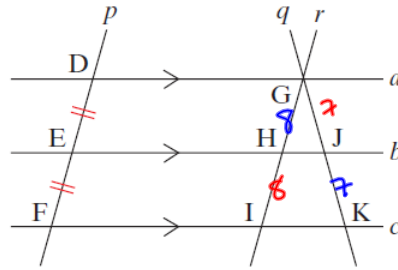
scale factor  $k = 7/10$

7.  $a, b$  and  $c$  are parallel lines.  
 $p, q$  and  $r$  are three transversals intersecting  $a, b$  and  $c$ .  
 $|DE| = |EF|, |GH| = 8$  cm and  $|JK| = 7$  cm.

Find (i)  $|HI|$  (ii)  $|GJ|$ .

$$\frac{|DE|}{|EF|} = \frac{1}{1} = \frac{|GH|}{|HI|} = \frac{8}{x}$$

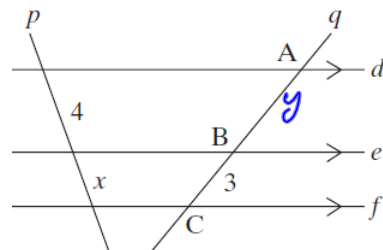
$$\Rightarrow x = 8$$



## Theorem 11

8. In the given figure,  $d, e$  and  $f$  are parallel lines.  
 $p$  and  $q$  are two transversals.  
 The transversal  $p$  is divided in the ratio  $4:x$ .  
 Find, in terms of  $x$ , the length of the line segment  $[AB]$ .

## Theorem 11



$$\frac{y}{3} = \frac{4}{x}$$

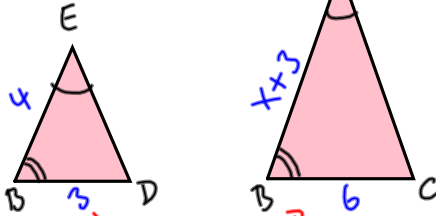
$$y = \frac{4(3)}{x}$$

$$|AB| = \frac{12}{x}$$

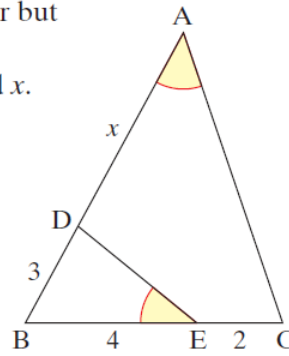
$$\text{let } |AB| = y$$

17. The triangles ABC and BED are similar but DE is not parallel to AC.  
Work out the length of the side marked x.

Rotate and flip  $\triangle DBE$



multiplier  $\times 2$



Similar triangles

$$\Rightarrow 2(4) = x + 3$$

$$x = 8 - 3$$

$$x = 5$$

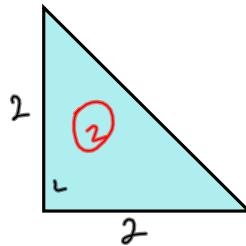
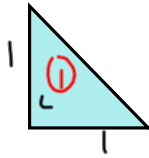
scale factor  $k = 2$

## Scale factor for area?

$$\Delta = \frac{Bh}{2}$$

consider the following

If  $k=2$

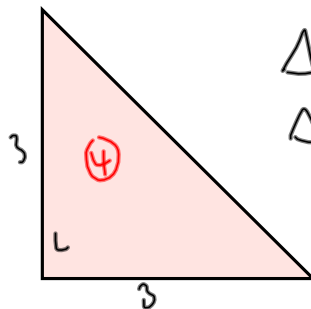
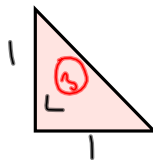


$$\Delta_{(1)} = \frac{(1)(1)}{2} = \frac{1}{2}$$

$$\Delta_{(2)} = \frac{(2)(2)}{2} = 2$$

$$\Delta_{(1)} : \Delta_{(2)} = 1 : 4$$

If  $k=3$

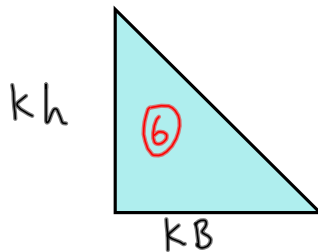
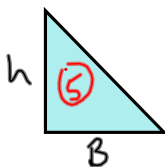


$$\Delta_{(3)} = \frac{(1)(1)}{2} = \frac{1}{2}$$

$$\Delta_{(4)} = \frac{(3)(3)}{2} = \frac{9}{2}$$

$$\Delta_{(3)} : \Delta_{(4)} = 1 : 9$$

If  $k=k$



$$\Delta = \frac{Bh}{2}$$

$$\Delta_{(5)} = \frac{Bh}{2}$$

$$\Delta_{(6)} = \frac{(kh)(kB)}{2} = \frac{k^2 Bh}{2}$$

$$\Delta_{(5)} : \Delta_{(6)} = 1 : k^2$$

⇒ scale factor for Area is  $k^2$

(where  $k$  is the scale factor for the sides.)