

# Geometry 1

chapter

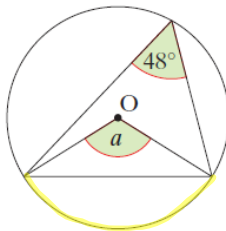
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## Section 3.4 Circle theorems

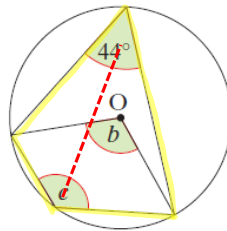
PROJECT MATHS – STRAND 2  
**Text & Tests** 4  
 LEAVING CERTIFICATE  
 HIGHER LEVEL

### Exercise 3.4

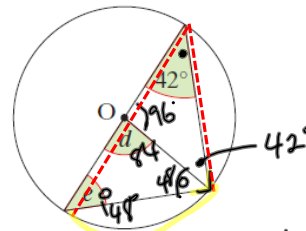
1. Find the measure of the angle marked with a letter in each of the following circles, where O is the centre.



$$\begin{aligned} a &= 2(48) \\ &= 96^\circ \end{aligned}$$

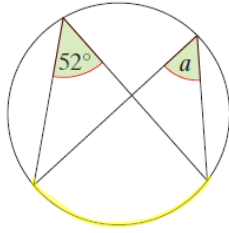


$$\begin{aligned} b &= 2(44) \\ &= 88^\circ \\ c &= 180 - 44 \\ &= 136^\circ \end{aligned}$$

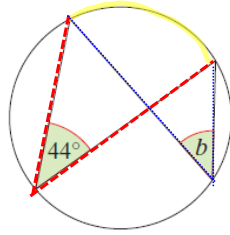


$$\begin{aligned} d &= 42 + 42 \\ &= 84^\circ \\ e &= \frac{96}{2} = 48^\circ \end{aligned}$$

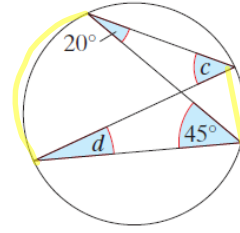
2. Find the measure of the angle marked with a letter in each of these circles:



$$a = 52^\circ$$



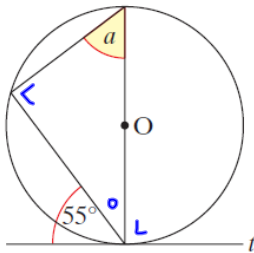
$$b = 44^\circ$$



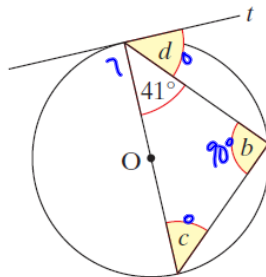
$$c = 45^\circ$$

$$d = 20^\circ$$

9. O is the centre of the given circles and  $t$  is a tangent in each case. Work out the size of the angles marked with a letter.



$$\Rightarrow a = 55^\circ$$

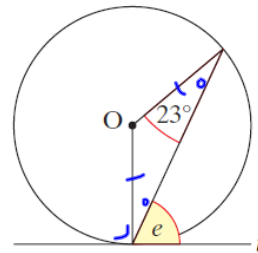


$$b = 90^\circ$$

$$c = d = 90 - 41$$

$$c = 49^\circ$$

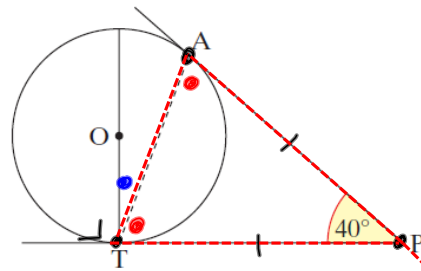
$$d = 49^\circ$$



$$e = 90 - 23$$

$$e = 67^\circ$$

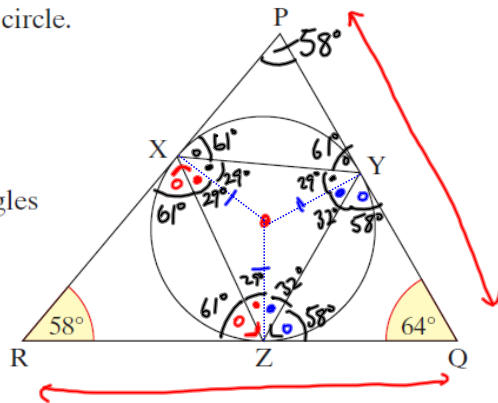
12. In the given diagram, PA and PT are tangents to the circle of centre O.  
If  $|\angle APT| = 40^\circ$ , find  $|\angle ATO|$ .



$$\bullet = \frac{180 - 40}{2} = 70$$

$$\bullet = 90 - 70 = 20^\circ$$

15. [RP], [PQ] and [QR] are tangents to the given circle.  
X, Y and Z are the points of contact.  
 $|\angle PRQ| = 58^\circ$  and  $|\angle PQR| = 64^\circ$ .
- Name three isosceles triangles.
  - Find  $|\angle PXY|$ .
  - Now find the measures of the interior angles of the triangle XYZ.

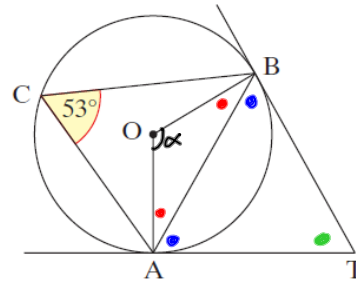


(i)  $\triangle PRQ$      $\triangle XYZ$      $\triangle PXY$   
 $\triangle RXZ$      $\triangle YZQ$

(ii)  $|\angle PXY| = 61^\circ$

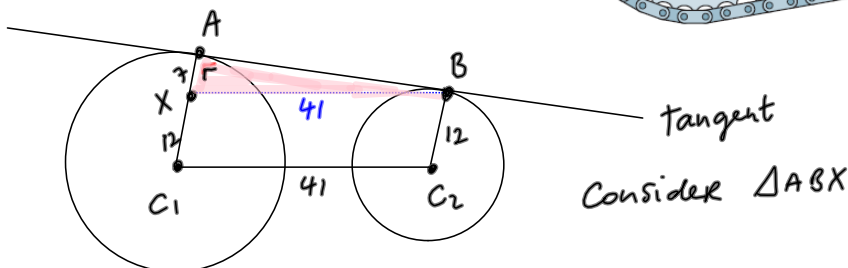
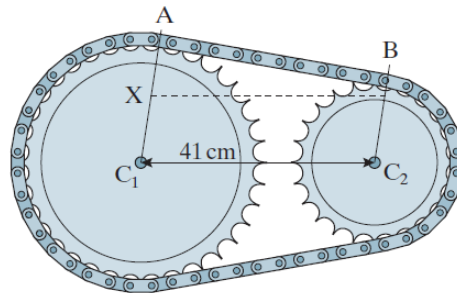
(iii)  $|\angle XZY| = 58^\circ$   
 $|\angle XYZ| = 61^\circ$   
 $|\angle ZXY| = 61^\circ$

17. ABC is a triangle inscribed in a circle with centre O.  
 TA and TB are tangents to the circle.  
 If  $|\angle ACB| = 53^\circ$ , find  
 (i)  $|\angle AOB|$   
 (ii)  $|\angle BTA|$   
 (iii)  $|\angle ABT|$ .



- (i)  $\alpha = 2(53) = 106^\circ$
- (ii)  $\bullet = 37^\circ$   
 $\bullet = 90 - 37 = 53^\circ$
- (iii)  $\bullet = 180 - 2(53) = 74^\circ$

23. A chain fits tightly over two cogs of radii 19 cm and 12 cm as shown in the diagram. Given that the centres of the cogs are 41 cm apart, calculate the length of the straight section of chain, [AB].  
 (The dotted line, parallel to  $[C_1C_2]$ , may be useful.)



$$a^2 = b^2 + c^2 \Rightarrow |AB| = \sqrt{41^2 - 7^2} = 4\sqrt{102} \approx 40.4 \text{ cm}$$