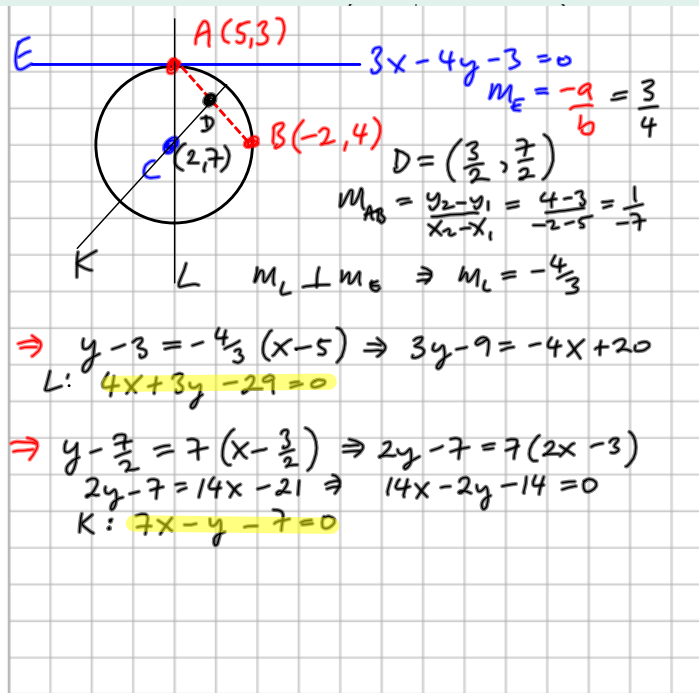**2. Equation of circle when given the equation of a tangent, the point of contact and one other point**

Example 3

Find the equation of the circle which touches the line $3x - 4y - 3 = 0$ at the point $A(5, 3)$ and which passes through the point $B(-2, 4)$.

$A(5,3)$

$3x - 4y - 3 = 0$

$m_E = \frac{-a}{b} = \frac{3}{4}$

Centre? $C = L \cap K$
Radius?

$B(-2,4)$

$D = \left(\frac{3}{2}, \frac{7}{2}\right)$

$C(2,7)$

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-3}{-2-5} = \frac{1}{-7}$

D is midpt $[AB]$

$m_K \perp m_{AB} \Rightarrow m_K = 7$

$m_L \perp m_E \Rightarrow m_L = -\frac{4}{3}$

$L: y - y_1 = m(x - x_1) \Rightarrow y - 3 = -\frac{4}{3}(x-5) \Rightarrow 3y - 9 = -4x + 20$

$L: 4x + 3y - 29 = 0$

$K: y - y_1 = m(x - x_1) \Rightarrow y - \frac{7}{2} = 7\left(x - \frac{3}{2}\right) \Rightarrow 2y - 7 = 7(2x - 3)$

$D \in K, m_K = 7$

$2y - 7 = 14x - 21 \Rightarrow 14x - 2y - 14 = 0$

$K: 7x - y - 7 = 0$

$C: L \cap K$

$L: 4x + 3y - 29 = 0$
$K: 7x - y - 7 = 0$
$4x + 3y = 29$
$21x - 3y = 21$
$25x = 50$
$x = 2$

$\Rightarrow 7(2) - y = 7 \Rightarrow 14 - y = 7$
$y = 7$

Centre $C. (2,7)$ $A = (5,3)$

Radius $= |AC|$

$r = \sqrt{(5-2)^2 + (3-7)^2} = \sqrt{25}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$r = 5$

equation
$(x-h)^2 + (y-k)^2 = r^2$

$(x-2)^2 + (y-7)^2 = 25$

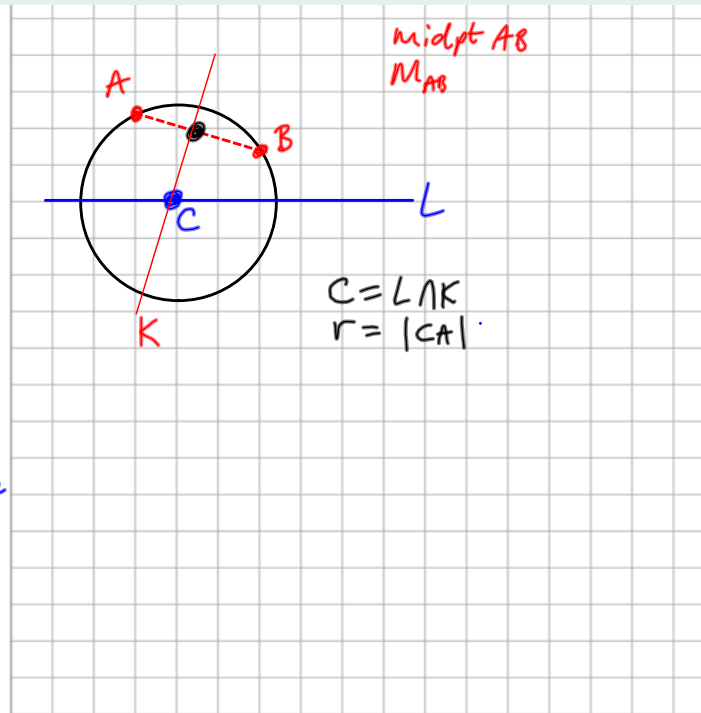**3. Equation of circle through two given points with its centre on a given line**

Example 4

Find the equation of the circle whose centre is on the line $l: 3x - y - 7 = 0$ and which passes through the points $A(1, 1)$ and $B(2, -1)$.

midpt AB

M_{AB}

Centre?

Radius?

$C = L \cap K$

$r = |CA|$

Circle

$(x-h)^2 + (y-k)^2 = r^2$

Example 5

Show that the line $x + 6y - 9 = 0$ is a tangent to the circle $x^2 + y^2 - 4x + 10y - 8 = 0$.

If tangent then $\Rightarrow$

$r = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$C(-g, -f)$
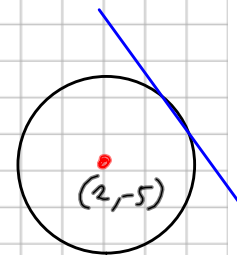
$r = \sqrt{g^2 + f^2 - c}$

$g = \dfrac{-4}{2} = -2$

$f = \dfrac{10}{2} = 5$

$c = -8$

$(2, -5)$

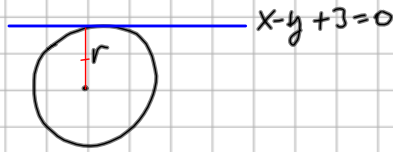$r = \sqrt{(-2)^2 + (5)^2 + 8}$

$r = \sqrt{4 + 25 + 8}$

$r = \sqrt{37}$

2. Write down the centre and the radius of the circle $(x-3)^2 + (y+4)^2 = 50$.
Hence show that the line $x - y + 3 = 0$ is a tangent to the circle.

$(x-h)^2 + (y-k)^2 = r^2$

C Centre $= (3,-4)$ Radius $= 5\sqrt{2}$

distance from C
to tangent is r

$d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

X-y +3 = 0

$d = \dfrac{|1(3) - 1(-4) + 3|}{\sqrt{1^2 + 1^2}} = \dfrac{10}{\sqrt{2}} = 5\sqrt{2}$

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4. The line $2x - 3y - 5 = 0$ is a tangent to a circle k.
If $(-1, 2)$ is the centre of k, find its equation.

$(x-h)^2 + (y-k)^2 = r^2$

k: $(x+1)^2 + (y-2)^2 = r^2$

radius?

$d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$r = \dfrac{|2(-1) - 3(2) - 5|}{\sqrt{2^2 + 3^2}} = \dfrac{|-13|}{\sqrt{13}} = \sqrt{13}$

$\Rightarrow$ K: $(x+1)^2 + (y-2)^2 = 13$