

2 Tangents to a circle parallel or perpendicular to a given line

**Example 2**

Find the equations of the two lines parallel to the line  $3x + 4y - 6 = 0$  and which are tangents to the circle  $x^2 + y^2 = 25$ .

// lines  $t$ :  $3x + 4y + k = 0$

distance from  $t$ 's to  $(0,0)$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$5 = \frac{|3(0) + 4(0) + k|}{\sqrt{3^2 + 4^2}}$$

$$25 = |k| \Rightarrow k = \pm 25$$

$t_1: 3x + 4y + 25 = 0$        $t_2: 3x + 4y - 25 = 0$

3. Tangents to a circle from a point P not on the circle

The perpendicular distance from the point  $(x_1, y_1)$  to the line  $ax + by + c = 0$  is

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

**Example 3**

Find the equations of the tangents to the circle  $x^2 + y^2 = 5$  from the point  $(5, 0)$ .

contains  $(5,0)$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = m(x - 5)$$

$$mx - y - 5m = 0$$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$\sqrt{5} = \frac{|m(0) - 1(0) - 5m|}{\sqrt{m^2 + 1}}$$

$$\sqrt{5} \sqrt{m^2 + 1} = |-5m|$$

$$(5)(m^2 + 1) = 25m^2$$

$$m^2 + 1 = 5m^2 \Rightarrow 4m^2 = 1 \Rightarrow m^2 = \frac{1}{4}$$

$$m = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2}$$

$$t_1: \frac{1}{2}x - y - 5(\frac{1}{2}) = 0 \Rightarrow x - 2y - 5 = 0$$

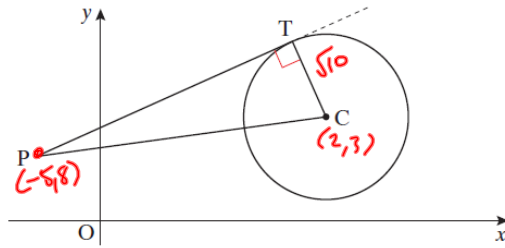
$$t_2: -\frac{1}{2}x - y - 5(-\frac{1}{2}) = 0 \Rightarrow x + 2y - 5 = 0$$

#### 4. Length of a tangent to a circle from a given point

To find the length of the tangent [PT],

$$|CP|^2 = |PT|^2 + |CT|^2$$

$$\Rightarrow |PT|^2 = |CP|^2 - |CT|^2$$



#### Example 4

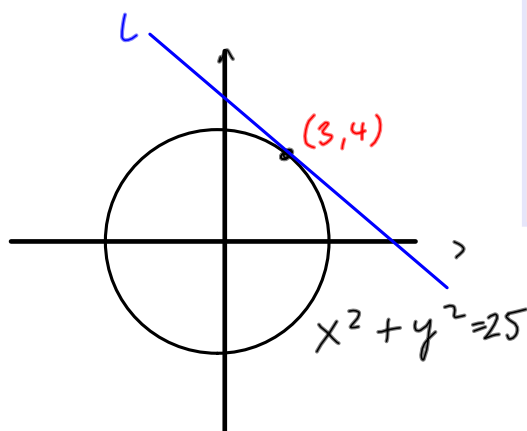
Find the length of the tangent from the point  $(-5, 8)$  to the circle  $x^2 + y^2 - 4x - 6y + 3 = 0$ .

Centre  $(2, 3)$   
 Radius  $= \sqrt{g^2 + f^2 - c}$   
 $R = \sqrt{2^2 + 3^2 - 3} = \sqrt{10}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$   
 $|PC| = \sqrt{(2 - (-5))^2 + (3 - 8)^2} = \sqrt{7^2 + (-5)^2} = \sqrt{49 + 25} = \sqrt{74}$

Pythagoras  $|PT| = \sqrt{(\sqrt{74})^2 - (\sqrt{10})^2} = \sqrt{64} = 8$

Finding the tangent to a circle with centre  $(0,0)$  at given point is very easy!



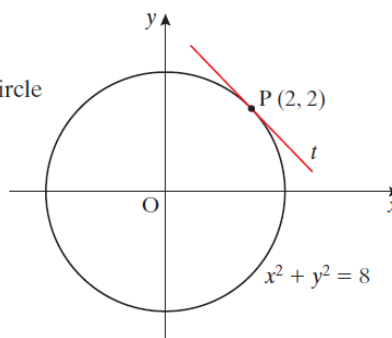
TANGENT FORMULA

$$x_1x + y_1y = r^2$$

$$\Rightarrow L: 3x + 4y - 25 = 0$$

## Exercise 4.4

1. Find the equation of the tangent,  $t$ , to the given circle  $x^2 + y^2 = 8$  at the point  $P(2, 2)$ .



Tangent  
 $x, x + y, y = r^2$   
 pt  $(2, 2)$

$$t: 2x + 2y - 8 = 0$$

$$\Rightarrow x + y - 4 = 0$$

2. Find the equation of the tangent to the circle  $x^2 + y^2 = 10$  at the point  $(-3, 1)$ .

$x, x + y, y = r^2$

Tangent at  $(-3, 1)$

$$-3x + y - 10 = 0$$

3. Find the equation of the tangent to the circle  $x^2 + y^2 = 17$  at the point  $(4, -1)$ .

$$x, X + y, Y = r^2$$

tangent at  $(4, -1)$

$$4x - y - 17 = 0$$