

Coordinate Geometry: The Circle

chapter

4

Section 4.5 Lines and circles: Common chord

PROJECT MATHS – STRAND 2
Text & Tests 4
LEAVING CERTIFICATE
HIGHER LEVEL

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1. Points of intersection of a line and a circle

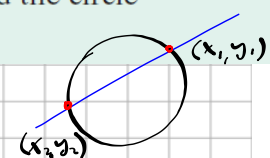
Example 1

Find the points of intersection of the line $x + 2y - 1 = 0$ and the circle $x^2 + y^2 + 2x + 8y - 8 = 0$.

① Rewrite linear

② Sub in & solve quadratic

③ Sub back into linear



$$x = 1 - 2y$$

$$(1 - 2y)^2 + y^2 + 2(1 - 2y) + 8y - 8 = 0$$

$$1 - 4y + 4y^2 + y^2 + 2 - 4y + 8y - 8 = 0$$

$$5y^2 - 5 = 0$$

$$y^2 = 1$$

$$\Rightarrow y = \pm 1$$

$$y = 1 \Rightarrow x = 1 - 2(1) = -1 \quad \text{pt } (-1, 1)$$

$$y = -1 \Rightarrow x = 1 - 2(-1) = 3 \quad \text{pt } (3, -1)$$

Example 2

Find the point(s) of intersection of the line $2x - y + 8 = 0$ and the circle $x^2 + y^2 + 4x + 2y = 0$ and hence show that the line is a tangent to the circle.

rewrite linear

$$2x + 8 = y$$

sub and solve

$$\begin{aligned} x^2 + (2x+8)^2 + 4x + 2(2x+8) &= 0 \\ x^2 + 4x^2 + 32x + 64 + 4x + 4x + 16 &= 0 \\ 5x^2 + 40x + 80 &= 0 \\ x^2 + 8x + 16 &= 0 \\ (x+4)(x+4) &= 0 \\ \Rightarrow x &= 4 \text{ and } -4 \end{aligned}$$

sub back

$$x = -4 \Rightarrow y = 2(-4) + 8 = 0$$

pt $(-4, 0)$

only one pt. of intersection
 \Rightarrow its a tangent.

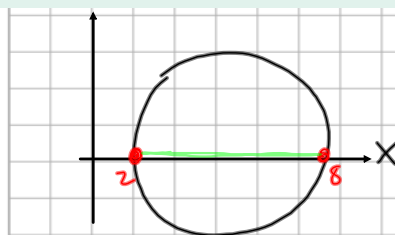
2. Where a circle intersects the axes

Example 3

Find the length of the intercept the circle $x^2 + y^2 - 10x + 8y + 16 = 0$ makes on the x-axis.

x-axis is $y=0$

sub into circle



$$\begin{aligned} x^2 + (0)^2 - 10x + 8(0) + 16 &= 0 \\ x^2 - 10x + 16 &= 0 \\ (x-8)(x-2) &= 0 \\ x &= 8 \text{ or } x = 2 \end{aligned}$$

distance $= 8 - 2 = 6$

The points of intersection of two circles may be found by following these steps:

- find the equation of the common chord
- find the points of intersection of the chord and any one of the circles.

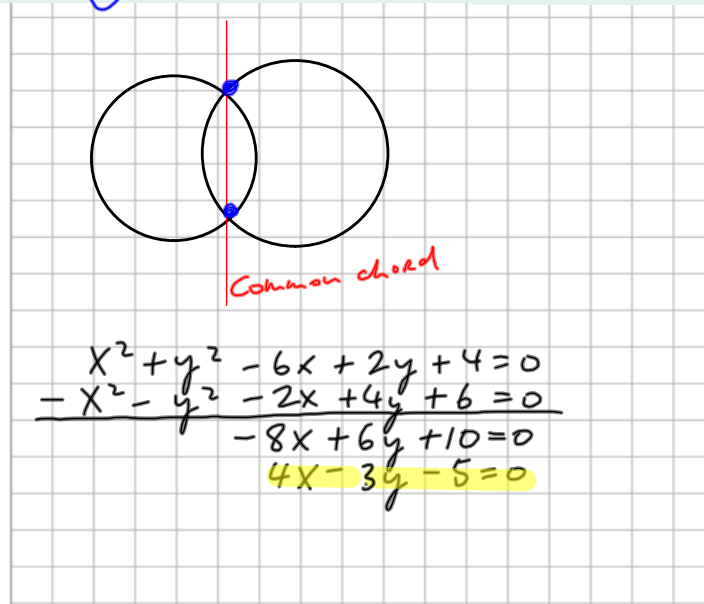
3. The common chord or common tangent of two circles

Example 4

Find the equation of the common chord of the two circles

$$S_1: x^2 + y^2 - 6x + 2y + 4 = 0 \quad \text{and} \quad S_2: x^2 + y^2 + 2x - 4y - 6 = 0.$$

$$S_1 - S_2 = \text{chord}$$



Exercise 4.5

- Find the points of intersection of the line $3x - y + 5 = 0$ and the circle $x^2 + y^2 = 5$.

Rewrite linear
sub in and
solve quadratic

$$\begin{aligned} y &= 3x + 5 \\ x^2 + (3x + 5)^2 &= 5 \\ x^2 + 9x^2 + 30x + 25 &= 5 \\ 10x^2 + 30x + 20 &= 0 \\ x^2 + 3x + 2 &= 0 \\ (x + 2)(x + 1) & \\ \Rightarrow x &= -2 \text{ or } -1 \end{aligned}$$

sub back in

$$\begin{aligned} y &= 3(-1) + 5 = -3 + 5 = 2 \\ y &= 3(-2) + 5 = -6 + 5 = -1 \\ \text{pt } &(-1, 2) \text{ and pt } (-2, -1) \end{aligned}$$