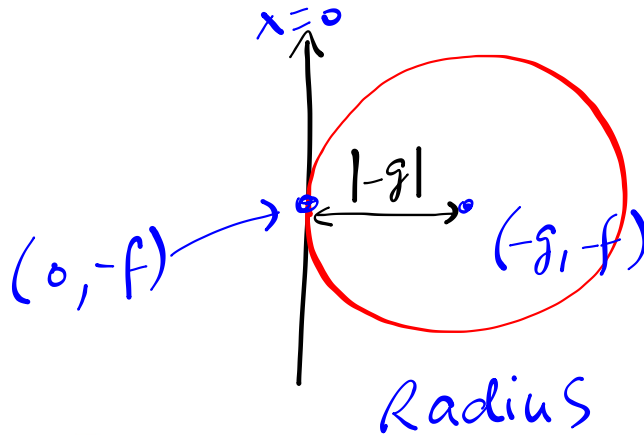


Coordinate Geometry: The Circle

chapter

4

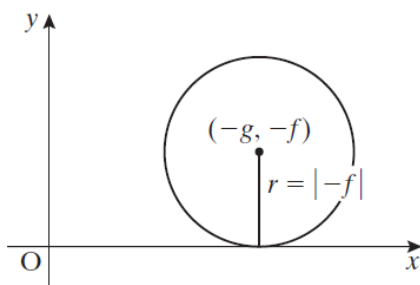
Section 4.7 Circles touching the x -axis or y -axis



PROJECT MATHS – STRAND 2
Text & Tests 4
LEAVING CERTIFICATE
HIGHER LEVEL

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1. Circle touching the x -axis



$$\text{Radius} = |-f|$$

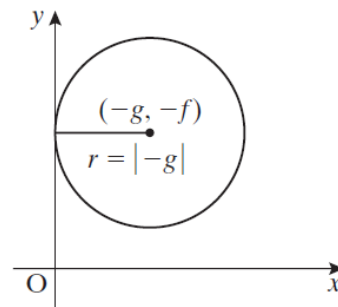
$$\Rightarrow \sqrt{g^2 + f^2 - c} = |-f|$$

$$\Rightarrow g^2 + f^2 - c = f^2$$

$$\Rightarrow g^2 - c = 0$$

$$\Rightarrow g^2 = c$$

2. Circle touching the y -axis



$$\text{Radius} = |-g|$$

$$\Rightarrow \sqrt{g^2 + f^2 - c} = |-g|$$

$$\Rightarrow g^2 + f^2 - c = g^2$$

$$\Rightarrow f^2 - c = 0$$

$$\Rightarrow f^2 = c$$

Example 1

Find the equations of the two circles which contain the points $(3, -2)$ and $(2, -1)$ and which touch the x -axis.

$x^2 + y^2 + 2gx + 2fy + c = 0$

Radius = $| -f | \Rightarrow g^2 = c$ ③

Sub in $(3, -2) \Rightarrow (3)^2 + (-2)^2 + 2g(3) + 2f(-2) + c = 0$
 $9 + 4 + 6g - 4f + c = 0$
 $6g - 4f + c = -13$ ①

Sub in $(2, -1) \Rightarrow (2)^2 + (-1)^2 + 2g(2) + 2f(-1) + c = 0$
 $4 + 1 + 4g - 2f + c = 0$
 $4g - 2f + c = -5$ ②

eliminate f

$① - 2② \Rightarrow$

$$\begin{array}{r} 6g - 4f + c = -13 \\ -8g + 4f - 2c = 10 \\ \hline -2g - c = -3 \end{array} \Rightarrow 2g + c = 3$$
 ④

Sub ③ into ④
 SOLVE QUADRATIC

$$\Rightarrow 2g + g^2 = 3 \Rightarrow g^2 + 2g - 3 = 0$$

$$(g + 3)(g - 1) = 0 \Rightarrow g = -3 \text{ or } g = 1$$

from diagram we see $-g$ is negative $\Rightarrow g = 1$ not -3

③ $\Rightarrow c = (1)^2$ and ② $4(1) - 2f + (1) = -5 \Rightarrow -2f = -10 \Rightarrow f = 5$
 $c = 1$

\Rightarrow Circle: $x^2 + y^2 + 2x + 10y + 1 = 0$