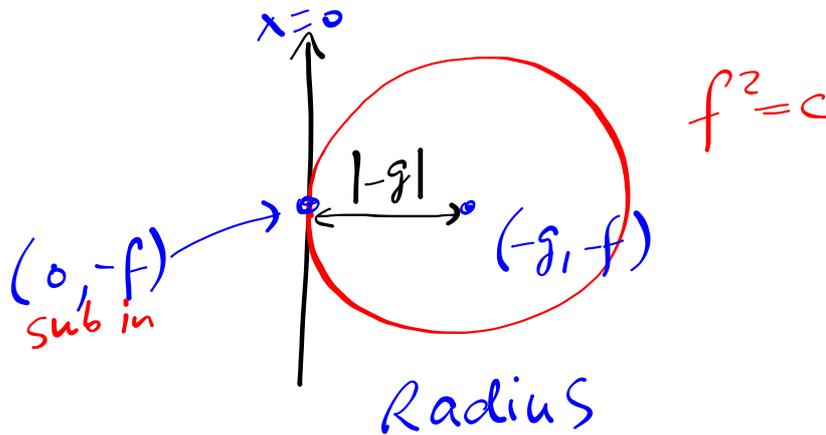


# Coordinate Geometry: The Circle

chapter

4

## Section 4.7 Circles touching the x-axis or y-axis



PROJECT MATHS - STRAND 2  
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### Example 1

Find the equations of the two circles which contain the points  $(3, -2)$  and  $(2, -1)$  and which touch the x-axis.

Diagram showing a circle touching the x-axis at  $(g, 0)$  with center  $(-g, f)$  and radius  $|f|$ . The general equation is  $x^2 + y^2 + 2gx + 2fy + c = 0$ . The radius is  $|f| \Rightarrow g^2 = c$  (3).

Sub in  $(3, -2) \Rightarrow (3)^2 + (-2)^2 + 2g(3) + 2f(-2) + c = 0$   
 $9 + 4 + 6g - 4f + c = 0$   
 $6g - 4f + c = -13$  (1)

Sub in  $(2, -1) \Rightarrow (2)^2 + (-1)^2 + 2g(2) + 2f(-1) + c = 0$   
 $4 + 1 + 4g - 2f + c = 0$   
 $4g - 2f + c = -5$  (2)

eliminate  $f$   
 $(1) - 2(2) \Rightarrow$   
 $6g - 4f + c = -13$   
 $-8g + 4f - 2c = 10$   
 $-2g - c = -3 \Rightarrow 2g + c = 3$  (4)

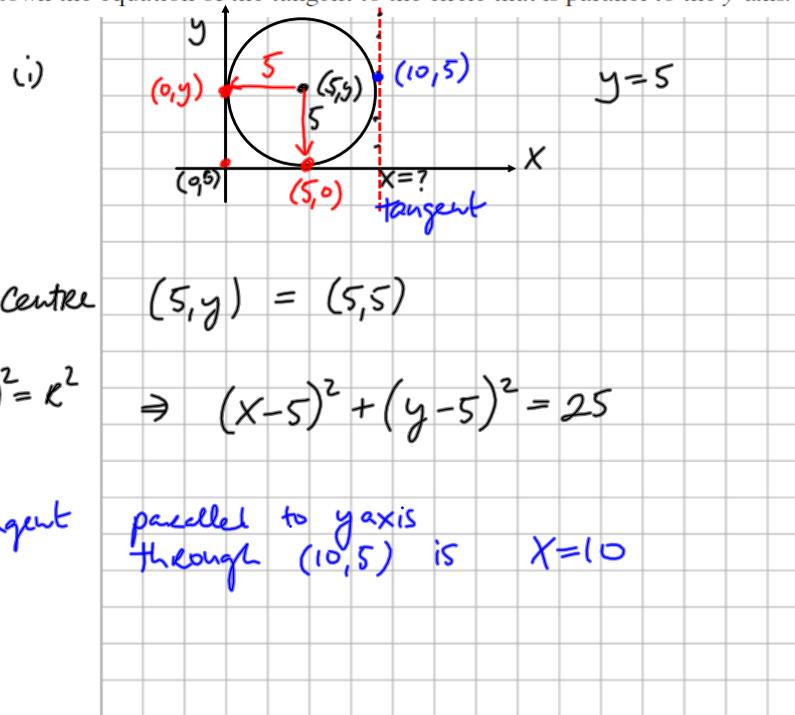
Sub (3) into (4)  
 SOLVE QUADRATIC  
 $\Rightarrow 2g + g^2 = 3 \Rightarrow g^2 + 2g - 3 = 0$   
 $(g + 3)(g - 1) = 0 \Rightarrow g = -3$  or  $g = 1$

from diagram we see  $-g$  is negative  $\Rightarrow g = 1$  not  $-3$

(3)  $\Rightarrow c = (1)^2$  and (2)  $4(1) - 2f + (1) = -5 \Rightarrow -2f = -10 \Rightarrow f = 5$   
 $c = 1$

$\Rightarrow$  Circle:  $x^2 + y^2 + 2x + 10y + 1 = 0$

3. The coordinates of the centre of a circle are  $(5, y), y > 0$ .  
 The  $x$ -axis and  $y$ -axis are both tangents to this circle.
- Write down the value of  $y$ .
  - Write down the equation of the circle.
  - Write down the equation of the tangent to the circle that is parallel to the  $y$ -axis.



7. The  $y$ -axis is a tangent to the circle  $x^2 + y^2 + 2gx + 2fy + c = 0$ .  
 Prove that  $f^2 = c$ .  
 A circle touches the  $y$ -axis at the point  $(0, -3)$  and it contains the point  $(4, 1)$ .  
 Find the equation of this circle.

