

(9) Derive  $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

use:

$$\tan A = \frac{\sin A}{\cos A}$$

$$\textcircled{5} \quad \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\textcircled{7} \quad \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$\tan(A+B) = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

÷ top and bottom  
by  $(\cos A \cos B)$

$$\tan(A+B) = \frac{\frac{\sin A \cancel{\cos B}}{\cancel{\cos A} \cancel{\cos B}} + \frac{\cancel{\cos A} \sin B}{\cancel{\cos A} \cancel{\cos B}}}{\frac{\cancel{\cos A} \cancel{\cos B}}{\cancel{\cos A} \cancel{\cos B}} - \frac{\sin A \cancel{\sin B}}{\cancel{\cos A} \cancel{\cos B}}}$$

$$\Rightarrow \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \quad \text{QED}$$

$$1. \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$2. \sec \theta = \frac{1}{\cos \theta}$$

$$3. \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$4. \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$5. \sin^2 \theta + \cos^2 \theta = 1$$

$$6. 1 + \tan^2 \theta = \sec^2 \theta$$

$$* \cos^2 A = 1 - \sin^2 A$$

### Example 1

Prove these identities:

(i)  $\sec A - \tan A \sin A = \cos A$

(ii)  $\tan \theta \sqrt{1 - \sin^2 \theta} = \sin \theta$

$$\begin{aligned} \text{(i)} &= \frac{1}{\cos A} - \frac{\sin A}{\cos A} \sin A \\ &= \frac{1 - \sin^2 A}{\cos A} \quad [\sin^2 \theta + \cos^2 \theta = 1] \\ &= \frac{\cos^2 A}{\cos A} \\ &= \cos A \quad \text{QED} \quad \text{☺} \end{aligned}$$

$$\begin{aligned} \text{(ii)} & \quad [\cos^2 \theta = 1 - \sin^2 \theta] \\ &= \frac{\sin \theta}{\cos \theta} \left( \sqrt{\cos^2 \theta} \right) \\ &= \frac{\sin \theta}{\cancel{\cos \theta}} \cancel{\cos \theta} \\ &= \sin \theta \end{aligned}$$

## Exercise 5.1

Prove the following identities: 1.  $\cos A \tan A = \sin A$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cos A \tan A = \cancel{\cos A} \frac{\sin A}{\cancel{\cos A}} = \sin A$$

$$1. \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$2. \sec \theta = \frac{1}{\cos \theta}$$

$$3. \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$4. \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$5. \sin^2 \theta + \cos^2 \theta = 1$$

$$6. 1 + \tan^2 \theta = \sec^2 \theta$$

Prove the following identities: 2.  $\sin \theta \sec \theta = \tan \theta$

$$\sec A = \frac{1}{\cos A}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\Rightarrow \sin \theta \sec \theta = \sin \theta \left( \frac{1}{\cos \theta} \right)$$

$$= \tan \theta$$

$$1. \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$2. \sec \theta = \frac{1}{\cos \theta}$$

$$3. \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$4. \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$5. \sin^2 \theta + \cos^2 \theta = 1$$

$$6. 1 + \tan^2 \theta = \sec^2 \theta$$

Prove the following identities:

3.  $\sin \theta \tan \theta + \cos \theta = \sec \theta$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\text{LCD} = \cos A$$

$$\sin^2 A + \cos^2 A = 1$$

$$\frac{1}{\cos A} = \sec A$$

$$\begin{aligned} \sin A \tan A + \cos A &= \sin A \frac{\sin A}{\cos A} + \cos A \\ &= \frac{\sin^2 A + \cos^2 A}{\cos A} \\ &= \frac{1}{\cos A} \\ &= \sec A \end{aligned}$$

1.  $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$

2.  $\sec \theta = \frac{1}{\cos \theta}$

3.  $\tan \theta = \frac{\sin \theta}{\cos \theta}$

4.  $\cot \theta = \frac{\cos \theta}{\sin \theta}$

5.  $\sin^2 \theta + \cos^2 \theta = 1$

6.  $1 + \tan^2 \theta = \sec^2 \theta$

Prove the following identities:

4.  $\frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}} = \tan \theta$

$$1 - \sin^2 \theta = \cos^2 \theta$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\begin{aligned} \Rightarrow \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}} &= \frac{\sin \theta}{\sqrt{\cos^2 \theta}} = \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \end{aligned}$$

1.  $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$

2.  $\sec \theta = \frac{1}{\cos \theta}$

3.  $\tan \theta = \frac{\sin \theta}{\cos \theta}$

4.  $\cot \theta = \frac{\cos \theta}{\sin \theta}$

5.  $\sin^2 \theta + \cos^2 \theta = 1$

6.  $1 + \tan^2 \theta = \sec^2 \theta$

Prove the following identities: 5.  $\sec A - \sin A \tan A = \cos A$

$$\sec A = \frac{1}{\cos A}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$1 - \sin^2 A = \cos^2 A$$

$$\begin{aligned} \sec A - \sin A \tan A &= \frac{1}{\cos A} - \frac{\sin A \sin A}{\cos A} \\ &= \frac{1 - \sin^2 A}{\cos A} \\ &= \frac{\cos^2 A}{\cos A} \\ &= \cos A \end{aligned}$$

1.  $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$

2.  $\sec \theta = \frac{1}{\cos \theta}$

3.  $\tan \theta = \frac{\sin \theta}{\cos \theta}$

4.  $\cot \theta = \frac{\cos \theta}{\sin \theta}$

5.  $\sin^2 \theta + \cos^2 \theta = 1$

6.  $1 + \tan^2 \theta = \sec^2 \theta$