Changing sums and differences to products

Example 2

Express as a product (i) $\cos 5A + \cos 3A$ (ii) $\sin 3A - \sin A$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$\cos 5A + \cos 3A$

$= 2\cos\left(\frac{5A+3A}{2}\right)\cos\left(\frac{5A-3A}{2}\right)$

$= 2\cos(4A)\cos(A)$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$\sin 3A - \sin A$

$= 2\cos\left(\frac{3A+A}{2}\right)\sin\left(\frac{3A-A}{2}\right)$

$= 2\cos(2A)\sin(A)$

Example 3

Show that $\dfrac{\boxed{\sin 3A} - \sin 2A + \boxed{\sin A}}{\boxed{\cos 3A} + \boxed{\cos A} - \cos 2A} = \tan 2A.$

Sum → product

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\Rightarrow \sin 3A + \sin A = 2\sin\left(\frac{3A+A}{2}\right)\cos\left(\frac{3A-A}{2}\right)$

$= 2\sin 2A\cos A$

Sum → product

$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\Rightarrow \cos 3A + \cos A = 2\cos\left(\frac{3A+A}{2}\right)\cos\left(\frac{3A-A}{2}\right)$

$= 2\cos 2A\cos A$

$LHS = \dfrac{2\sin 2A\cos A - \sin 2A}{2\cos 2A\cos A - \cos 2A}$

factorise

$= \dfrac{\sin 2A\,\cancel{(2\cos A - 1)}}{\cos 2A\,\cancel{(2\cos A - 1)}} = \tan 2A$