

3. Without using a calculator, show that each of the following is true:

(i)  $\sin 75^\circ - \sin 15^\circ = \frac{1}{\sqrt{2}}$

(ii)  $\sin 10^\circ + \sin 80^\circ = \sqrt{2} \cos 35^\circ$ .

(i)

Log Tables

If  $A+B=75^\circ$   
and  $A-B=15^\circ$

$\Rightarrow 2A = 90^\circ$

$A = 45^\circ$

$\Rightarrow B = 30^\circ$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$\sin 75^\circ - \sin 15^\circ = \sin(45^\circ + 30^\circ) - \sin(45^\circ - 30^\circ)$$

$$= 2 \cos 45^\circ \sin 30^\circ$$

$$= 2 \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{2}\right) = \frac{1}{\sqrt{2}}$$

QED

(ii)

Log Tables

If  $A+B=80^\circ$

$A-B=10^\circ$

$2A = 90^\circ$

$A = 45^\circ$

$\Rightarrow B = 35^\circ$

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$\sin 10^\circ + \sin 80^\circ = \sin 80^\circ + \sin 10^\circ$$

$$= \sin(45^\circ + 35^\circ) + \sin(45^\circ - 35^\circ)$$

$$= 2 \sin 45^\circ \cos 35^\circ$$

$$= 2 \left(\frac{1}{\sqrt{2}}\right) \cos 35^\circ$$

$$= \sqrt{2} \cos 35^\circ$$

QED