

chapter

3

## Probability 2

## Section 3.4 How to show events are independent

Independent if :

- ①  $P(A \cap B) = P(A) \times P(B)$
- ②  $P(A) \times P(B) = P(A) \times P(B|A)$   
 $P(B) \times P(A) = P(B) \times P(A|B)$
- ③  $P(A) = P(A|B)$   
 $P(B) = P(B|A)$

PROJECT MATHS  
**Text & Tests 5**  
 LEAVING CERTIFICATE  
 HIGHER LEVEL  
 STRAND 1  
 PROBABILITY & STATISTICS

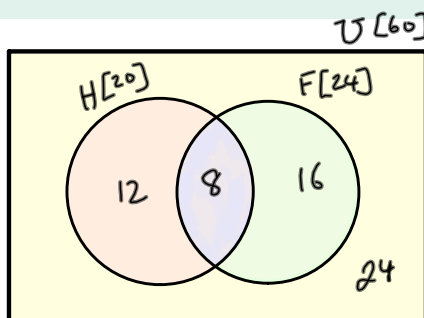
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## Example 1

In a group of 60 students, 20 study History, 24 study French, 8 study both History and French and 24 study neither.

Illustrate this information on a Venn diagram.

Now investigate if the events 'a student studies History' and 'a student studies French' are independent.



$$P(H) = \frac{20}{60}$$

$$P(F) = \frac{24}{60}$$

$$P(F \cap H) = \frac{8}{60}$$

Test is  $P(F \cap H) \stackrel{?}{=} P(H) \cdot P(F)$

$$\frac{8}{60} \stackrel{?}{=} \left(\frac{20}{60}\right) \left(\frac{24}{60}\right) = \left(\frac{1}{3}\right) \left(\frac{2}{5}\right) = \frac{2}{15} = \frac{8}{60} \checkmark$$

yes

### Example 2

Two events A and B are such that  $P(A) = 0.5$ ,  $P(B) = 0.4$  and  $P(A|B) = 0.3$ .

- Find  $P(A \cap B)$ .
- Investigate whether or not the events A and B are independent.

$$(i) \quad P(A \cap B) = ? \quad \begin{aligned} P(A \cap B) &= P(A) \cdot P(B|A) \\ &= P(B) \cdot P(A|B) \\ &= (0.4)(0.3) = 0.12 \end{aligned}$$

$$(ii) \quad \text{If Independent} \quad P(A) = P(A|B)$$

$$0.5 \neq 0.3$$

$$\Rightarrow \text{not independent}$$

### Example 3

Two ordinary fair dice, one red and one blue, are to be rolled once.

- Find the probability of the following events:  
 Event A: the number showing on the red dice will be a 5 or a 6.  
 Event B: the total of the numbers showing on the two dice will be 7.  
 Event C: the total of the numbers showing on the two dice will be 8.
- Show that events A and B are independent.
- Investigate if events A and C are independent.

$$P(\text{Event A}) = \frac{2}{6} = \frac{1}{3}$$

$$P(\text{Event B}) = \frac{1}{6}$$

$$P(\text{Event C}) = \frac{5}{36}$$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

- Show Events A and B are independent?

$$P(A \cap B) = \frac{2}{36} = \frac{1}{18} \quad \text{Is } P(A) \times P(B) = P(A \cap B)?$$

$$\frac{1}{3} \times \frac{1}{6} = \frac{1}{18} \quad \checkmark \Rightarrow \text{Independent.}$$

- Are A and C independent?

$$P(A) = \frac{1}{3} \quad P(C) = \frac{5}{36} \quad P(A) \times P(C) = \frac{5}{108}$$

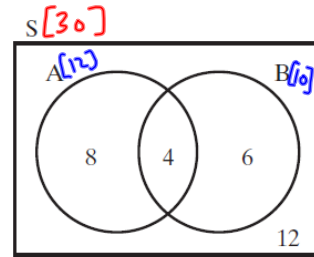
$$P(A \cap C) = \frac{2}{36} = \frac{1}{18} \neq \frac{5}{108} \Rightarrow \text{not independent}$$

## Exercise 3.4

1. The given Venn diagram shows the number of elements in the different regions.

Find (i)  $P(A)$  (ii)  $P(B)$  (iii)  $P(A \cap B)$ .

Hence show that A and B are independent events.



$$(i) P(A) = \frac{12}{30} = \frac{2}{5}$$

$$(ii) P(B) = \frac{10}{30} = \frac{1}{3}$$

$$(iii) P(A \cap B) = \frac{4}{30} = \frac{2}{15}$$

Independent if  $P(A) \times P(B) = P(A \cap B)$   
 $P(A) \times P(B) = \left(\frac{2}{5}\right)\left(\frac{1}{3}\right) = \frac{2}{15} = P(A \cap B)$   
 $\Rightarrow$  yes they're independent.