Exercise 3.5

1. A hand of four cards is dealt at random from a normal pack of 52 cards.
Find the probability that the hand contains
(i) exactly two queens (ii) four spades
(iii) four red cards (iv) four cards of the same suit.

$$\text{Probability} = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$\text{Total outcomes} = \binom{52}{4} = 270{,}725$$

$$P(Q,Q,F,F) = \left(\tfrac{4}{52}\right)\left(\tfrac{3}{51}\right)\left(\tfrac{48}{50}\right)\left(\tfrac{47}{49}\right) \times \binom{4}{2} = 0.025$$

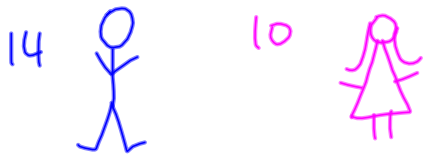
$$P(S,S,S,S) = \left(\tfrac{13}{52}\right)\left(\tfrac{12}{51}\right)\left(\tfrac{11}{50}\right)\left(\tfrac{10}{49}\right) = \frac{11}{4165}$$

$$P(R,R,R,R) = \left(\tfrac{26}{52}\right)\left(\tfrac{25}{51}\right)\left(\tfrac{24}{50}\right)\left(\tfrac{23}{49}\right) = \frac{46}{833}$$

$$P(HHHH) = \left(\tfrac{13}{52}\right)\left(\tfrac{12}{51}\right)\left(\tfrac{11}{50}\right)\left(\tfrac{10}{49}\right) = \frac{11}{4165}$$

$$P(\text{same suit}) = 4\left(\frac{11}{4165}\right) = \frac{44}{4165}$$

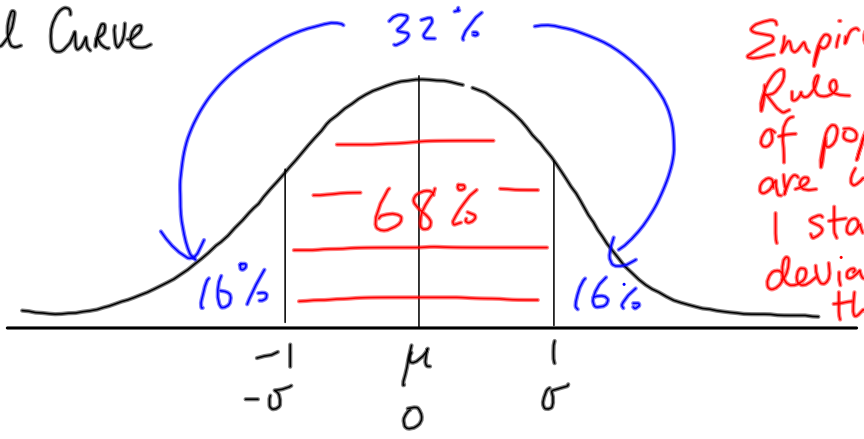
6. In a class of 24 students, there are 14 boys and 10 girls.
In a particular week three students celebrate their birthdays.
What is the probability that these three students
(i) are three boys or three girls
(ii) have their birthdays falling on different days of the week?



$$P(\text{3 same gender}) = P(\text{all boys}) \text{ or } P(\text{all girls}) = \frac{\binom{14}{3} + \binom{10}{3}}{\binom{24}{3}} = \frac{11}{46}$$

$$P(\text{different days}) = \binom{1}{1}\left(\tfrac{6}{7}\right)\left(\tfrac{5}{7}\right) = \frac{30}{49}$$

Normal Curve



Empirical Rule 68% of population are within 1 standard deviation of the mean.

6th year girls height = 155cm 160cm 165cm

P(random 6th yr is less than 155cm)? = 16% = 0.16