Example 2

Find the range of values of k for which the equation $x^2 + (k - 4)x + (k - 1) = 0$ has real roots.

$\Delta = b^2 - 4ac \geq 0$

$a = 1$

$b = k-4$

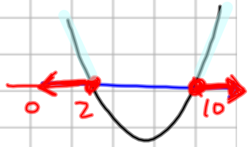
$c = k-1$

$$\Delta = (k-4)^2 - 4(k-1)(1)$$
$$= k^2 - 8k + 16 - 4k + 4$$
$$= k^2 - 12k + 20 \geq 0$$

If $k^2 - 12k + 20 = 0$

$(k-10)(k-2) = 0$

$k = 10 \quad , \quad k = 2$



$2 \geq k \geq 10$

2. Rational inequalities

Is this negative?

Example 3

Find the range of values of x for which $\dfrac{2x+1}{x+2} < \dfrac{1}{2}$.

If $\dfrac{2x+1}{x+2} = \dfrac{1}{2}$

$4x + 2 = x + 2$

$3x = 0$

$x = 0$

as this isn't given 2 values we need another method

$* \ (2)(x+2)$

$-x, -2$

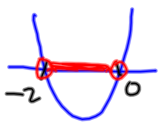
trick!

$* \ (2)(x+2)^2$

$$\frac{(2x+1)\,2(x+2)^{\cancel{2}}}{\cancel{(x+2)}} < \frac{1}{\cancel{2}}\cancel{(2)}(x+2)^2$$

$$4x^2 + 8x + 2x + \cancel{4} < x^2 + 4x + \cancel{4}$$
$$3x^2 + 6x < 0$$
$$x^2 + 2x < 0$$
$$x(x+2) < 0$$

If $x(x+2) = 0 \Rightarrow x = 0 \ , \ x = -2$



$\Rightarrow -2 < x < 0$

9. Solve each of the following rational inequalities for x.

(i) $\frac{x+3}{x+2} < 2, x \neq -2$ (ii) $\frac{x+5}{x-3} > 1, x \neq 3$ (iii) $\frac{2x-1}{x+3} > 3, x \neq -3$

trick

$* \ (x-3)^2 \Rightarrow$

this means that we get rid of fraction by multiplying by a positive number so we know inequality is still true.

$$\frac{x+5}{x-3} > 1$$

$$\frac{(x+5)(x-3)^{\cancel{2}}}{\cancel{(x-3)}} > 1(x-3)^2$$

$$\cancel{x^2} - 3x + 5x - 15 > \cancel{x^2} - 6x + 9$$

$$2x - 15 > -6x + 9$$

$$8x > 24$$

$$x > 3$$