

10. Given that $a^2 + b^2 \geq 2ab$, deduce an expression for (i) $a^2 + c^2$ and (ii) $c^2 + d^2$.
 Use these results to prove that $a^2 + b^2 + c^2 \geq ab + bc + ca$ for all real values of a, b and c .

Should be $c^2 + b^2$

$a^2 + c^2 = ?$ $c^2 + d^2 = ?$ $c^2 + b^2$	$\Rightarrow a^2 + c^2 \geq 2ac$ $\Rightarrow c^2 + d^2 \geq 2cd$ ← useless $\Rightarrow c^2 + b^2 \geq 2cb$
LHSs \geq RHSs \Rightarrow \therefore	$a^2 + b^2 + a^2 + c^2 + c^2 + b^2 \geq 2ab + 2ac + 2cb$ $2a^2 + 2b^2 + 2c^2 \geq 2ab + 2ac + 2cb$ $a^2 + b^2 + c^2 \geq ab + bc + ca$ QED.

11. If $p > 0$ and $q > 0$ and $p \neq q$, prove that $\frac{p+q}{2} > \sqrt{pq}$.

$\times 2$ square $-4pq$	$p+q > 2\sqrt{pq}$ $(p+q)^2 > (2\sqrt{pq})^2$ $p^2 + 2pq + q^2 > 4pq$ $p^2 - 2pq + q^2 > 0$ $(p-q)^2 > 0$ true
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18. Prove that $\sqrt{ab} > \frac{2ab}{a+b}$ if a and b are positive and unequal.

$$\times (a+b)$$

Square

$$- 4ab$$

$$(a+b)\sqrt{ab} > 2ab$$

$$(a+b)^2(\sqrt{ab})^2 > (2ab)^2$$

$$(a^2 + 2ab + b^2)(\cancel{ab}) > 4(\cancel{ab})(\cancel{ab})$$

$$a^2 - 2ab + b^2 > 0$$

$$(a-b)^2 > 0$$

True