chapter 7 **Algebra 3**

Section 7.9 Logarithmic function

$b^n = a \Leftrightarrow n = \log_b a$

$\log_{\square}\square$

$\log_{10}$

$\ln = \log_e$ natural log

eg. $8 = 2^n$, $\log_2 8 = 3$

$\log_{10} 1000 = 3$

$\log_4 2 = \frac{1}{2}$

$\log 1 = 0$

$\ln e = 1$

$\log 10 = 1$

$\log_a 9 = 1$

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The Laws of Logs

The laws of exponents lead to the following laws of logarithms. Here we assume x and y are positive real numbers.

1. $\log_a(xy) = \log_a(x) + \log_a(y)$
2. $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$
3. $\log_a(x^r) = r\log_a(x)$ for any real number r.

The Laws of Indices

$a^n \cdot a^m = a^{n+m}$

$(a^n)^m = a^{mn}$

$\frac{a^n}{a^m} = a^{n-m}$

1. The laws of logarithms

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Example 2

Without using a calculator, simplify the following number:

$$2\log_{10}3 + \log_{10}16 - 2\log_{10}\left(\frac{6}{5}\right)$$

$2\log 3 + \log 4^2 - 2\log\left(\frac{6}{5}\right)$

$2\log 3 + 2\log 4 - 2\log\frac{6}{5}$

$2\left[\log 3 + \log 4 - \log\frac{6}{5}\right]$

$2\left[\log\frac{(3)(4)}{\left(\frac{6}{5}\right)}\right]$

$= 2\log_{10}10 = 2(1) = 2$

6. Write each of the following in the form $\log_a x$ and then simplify:

(i) $\log_3 2 + 2\log_3 3 - \log_3 18$ (ii) $\log_8 72 - \log_8\left(\frac{9}{8}\right)$

$2\log_3 3$ (i)

$= \log_3 3^2 = \log_3 9$

but $3^0 = 1$

$\log_3 2 + 2\log_3 3 - \log_3 18$

$= \log_3 2 + \log_3 9 - \log_3 18$

$= \log_3\frac{(2)(9)}{18} = \log_3 1 = 0$

$64 = 8^2$ (ii)

$\log_8 72 - \log_8\left(\frac{9}{8}\right)$

$= \log_8\frac{72}{\left(\frac{9}{8}\right)} = \log_8 64 = 2$

$\log x + \log y = \log xy$

$\log x - \log y = \log\frac{x}{y}$

$\log x^n = n\log x$