Differential Calculus

chapter 2

Section 2.1 Average rate of change

PROJECT MATHS
Text & Tests 7

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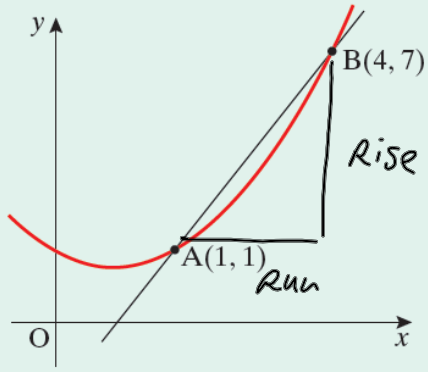
Example 1

Find the average rate of change of y with respect to x for the function $y = f(x)$ over the interval $[1, 4]$ as shown.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7-1}{4-1} = \frac{6}{3} = 2$$

Example 2

The temperature T (°C) in a classroom on a particular day can be modelled by the equation

$$T = \frac{200}{t^2 + 2t + 20}, \text{ where } t \text{ is the time after 6.00 p.m..}$$ t in hours

Find (i) the temperature in the room at 6.00 p.m.
(ii) the temperature in the room at midnight
(iii) the average rate of change of temperature from 6.00 p.m. to midnight.

(i) $t=0 \Rightarrow T = \frac{200}{0^2+2(0)+20} = \frac{200}{20} = 10\,°C$

(ii) $t=6 \Rightarrow T = \frac{200}{6^2+2(6)+20} = \frac{50}{17}\,°C \approx 2.9°C$

(iii) $\frac{\text{change in } T}{\text{time}} = \frac{\left(10 - \frac{50}{17}\right)}{6-0} = \frac{\left(\frac{120}{17}\right)}{6} = \frac{20}{17} \approx 1.2\ °C/h$

4 The depth, d cm, of water in a bath tub t minutes after the tap is turned on is modelled by the function $d(t) = \frac{-300}{(t+6)} + 50, t \geqslant 0$.
Find the average rate of change of the depth of the water in the tub over the first 10 minutes after the tap is turned on.

Start, $t=0 \Rightarrow$ | $d = \frac{-300}{(0+6)} + 50 = 0$ cm

after 10 mins, $t=10 \Rightarrow$ | $d = \frac{-300}{(10+6)} + 50 = 31.25$ cm

average rate of change $= \frac{\text{change}}{\text{time}}$ | $= \frac{31.25-0}{10} = 3.125$ cm/min