Differential Calculus

chapter 2

Section 2.3 Differentiation by rule

PROJECT MATHS
Text & Tests 7

64

In the previous section, we found the derivative of a function from first principles. This method becomes very tedious as the function becomes more complex.

We will now show how certain rules can be used to find the derivative of a function without having to use the method of differentiation from first principles.

It can be shown from first principles that if

(i) $y = x^2$, then $\frac{dy}{dx} = 2x$

(ii) $y = 4x^2$, then $\frac{dy}{dx} = 8x$

(iii) $y = x^3$, then $\frac{dy}{dx} = 3x^2$

The pattern in these derivatives suggests a general result which is given below:

Differentiation by rule

1. If $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$ for all $n \in \mathbb{R}$.
2. If $y = ax^n$, then $\frac{dy}{dx} = nax^{n-1}$

Example 1

Find the derivative of $f(x) = 6x^3 - 3x^2 + 4x$.
Hence, find $f'(2)$ and interpret the result.

$f(x) \to f'(x)$

$ax^n \to anx^{n-1}$

ADDITION Rule

$f'(x) = 18x^2 - 6x + 4$

$f'(2) = 18(2)^2 - 6(2) + 4 = 64$

Example 2

Find $\frac{dy}{dx}$ for each of the following:

$\frac{1}{x^1} = x^{-1}$

$\sqrt{x} = x^{\frac{1}{2}}$

$\frac{1}{x^2} = x^{-2}$

(i) $y = 3x^2 + \frac{2}{x} = 3x^2 + 2x^{-1}$

$\frac{dy}{dx} = 6x - 2x^{-2}$

(ii) $y = \sqrt{x} - \frac{4}{x^2} = x^{\frac{1}{2}} - 4x^{-2}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} + 8x^{-3}$