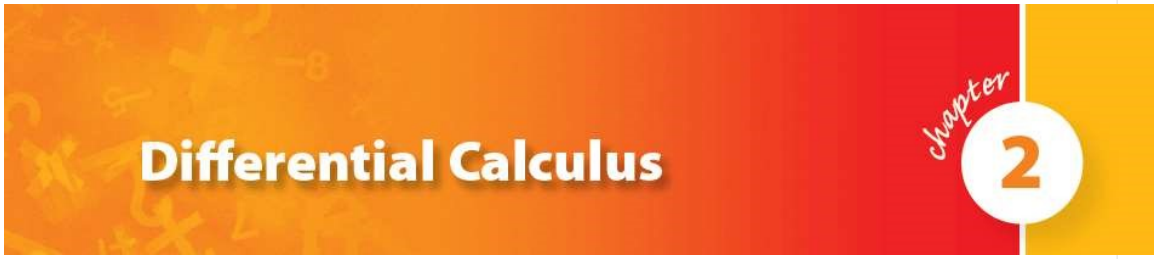

Section 2.6 The derivatives of trigonometric functions

Differentiation

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
e^{ax}	ae^{ax}
a^x	$a^x \ln a$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\tan x$	$\sec^2 x$
$\cos^{-1}\frac{x}{a}$	$-\frac{1}{\sqrt{a^2-x^2}}$
$\sin^{-1}\frac{x}{a}$	$\frac{1}{\sqrt{a^2-x^2}}$
$\tan^{-1}\frac{x}{a}$	$\frac{a}{a^2+x^2}$

trig. functions

$\frac{d}{dx}(\sin x) = \cos x$ $\frac{d}{dx}(\cos x) = -\sin x$ $\frac{d}{dx}(\tan x) = \sec^2 x$

Example 1

Differentiate each of the following with respect to x:
(i) $y = 3\sin x + 2\cos x$ (ii) $y = x^2 \sin x$

(i) $y = 3\sin x + 2\cos x$

$\frac{dy}{dx} = 3(\cos x) + 2(-\sin x) = 3\cos x - 2\sin x$

(ii) $y = x^2 \cdot \sin x$ (u, v)

PRODUCT

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$u = x^2$ $\frac{du}{dx} = 2x$

$v = \sin x$ $\frac{dv}{dx} = \cos x$

$\frac{dy}{dx} = x^2(\cos x) + (\sin x)(2x)$

$= x^2\cos x + 2x\sin x$

Example 2

Find the derivative of each of the following:
(i) $\cos(7x - 3)$ (ii) $\tan^2 3x$ (iii) $\sin^3(x^2 + 2)$

$f(x)$ $f'(x)$

$\cos x \to -\sin x$

Chain Rule

outside: $\cos u$

inside: $7x-3$

$f(x) = \cos(7x-3)$

$f'(x) = [-\sin(7x-3)] \cdot (7)$

$= -7\sin(7x-3)$

$f(x) \to f'(x)$

$\tan x \to \sec^2 x$

outside: u^2

middle: $\tan v$

inside: $3x$

$f'(x) = (\tan 3x)^2$

$f'(x) = 2(\tan 3x)' \cdot (\sec^2 3x) \cdot (3)$

$= 6\tan 3x \cdot \sec^2 3x$

$f(x) \to f'(x)$

$\sin x \quad \cos x$

outside: u^3

middle: $\sin v$

inside: x^2+2

$f(x) = \sin^3(x^2+2) = (\sin(x^2+2))^3$

$f'(x) = 3(\sin(x^2+2))^2 \cdot (\cos(x^2+2)) \cdot 2x$

$= 6x\sin^2(x^2+2) \cdot \cos(x^2+2)$

Example 3

If $f(x) = \frac{1 + \sin x}{\cos x}$, show that $f'(x) = \frac{1 + \sin x}{\cos^2 x}$ and hence evaluate $f(\pi)$.

Quotient

$u = 1 + \sin x$

$\frac{du}{dx} = \cos x$

$v = \cos x$

$\frac{dv}{dx} = -\sin x$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\cos^2 x + \sin^2 x = 1$

$$f'(x) = \frac{(\cos x)(\cos x) - (1+\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1 + \sin x}{\cos^2 x}$$

$$f'(\pi) = \frac{1 + \sin \pi}{\cos^2 \pi} = \frac{1+0}{(1)^2} = 1$$