chapter 3 Applications of Differential Calculus

Section 3.1 Tangents – Increasing and decreasing functions

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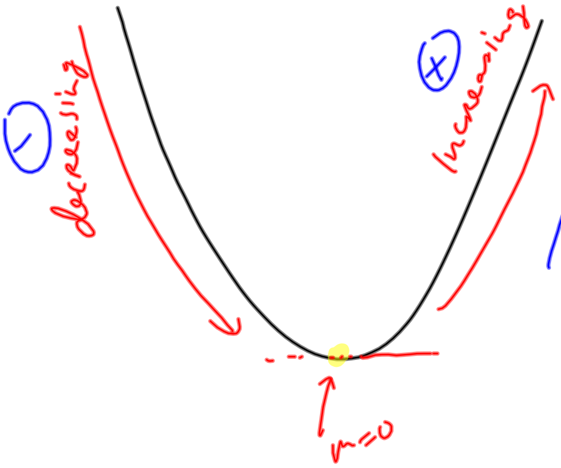
The Derivative is the Slope!

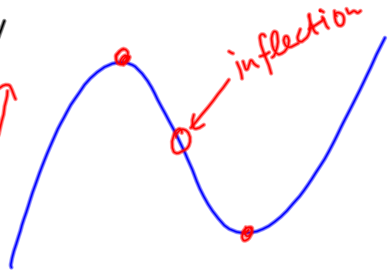
$$m = \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

Δ = change in

Key words

increasing function **decreasing function** **stationary points** **local maximum** **local minimum** **point of inflection** **slope function** **rate of change** **velocity** **acceleration** **related rates of change**





In the previous chapter, it was stated that for any function $y = f(x)$, the derived function $\frac{dy}{dx}$ can be interpreted geometrically as the slope of the tangent to the curve at any point on the curve.

If $y = f(x)$, then $y = f'(a)$ gives the slope of the tangent to the curve at the point where $x = a$.

Thus, to find the equation of the tangent to a curve at the point (x_1, y_1):

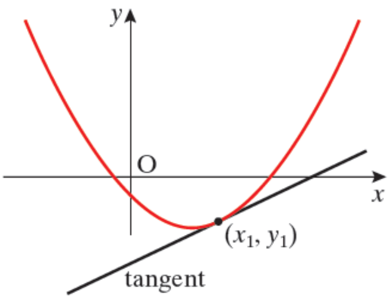
(i) Find $\frac{dy}{dx}$.

(ii) Evaluate $\frac{dy}{dx}$ at (x_1, y_1).

(iii) Use the equation

$$y - y_1 = m(x - x_1)$$

to find the equation of the tangent at the point (x_1, y_1).



Example 1

Find the equation of the tangent to the curve $y = \frac{x^2}{3} - x + 1$ at the point $(4, \frac{7}{3})$.

x_1, y_1

Slope = ?

$\frac{dy}{dx} = \frac{2}{3}x - 1$

when $x = 4$ $\Rightarrow$

$\left.\frac{dy}{dx}\right|_{x=4} = \frac{2}{3}(4) - 1 = \frac{8}{3} - 1 = \frac{5}{3}$

tangent?

$y - y_1 = m(x - x_1)$

$y - \frac{7}{3} = \frac{5}{3}(x - 4)$

×3

$3y - 7 = 5x - 20$

$5x - 3y - 13 = 0$

Example 2

At what points on the curve $y = x^3 - 9x^2 + 20x - 8$ is the tangent parallel to the line $4x + y - 3 = 0$?

Slope of $4x + y - 3 = 0$?

$a = 4$, $b = 1$

$m = -\frac{a}{b}$

$m = \frac{-4}{1} = -4$

Slope of curve

$\frac{dy}{dx} = 3x^2 - 18x + 20$

// $\Rightarrow$ equal slopes $\Rightarrow$

$3x^2 - 18x + 20 = -4$

$3x^2 - 18x + 24 = 0$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

$x = 4$ or $x = 2$

$x = 4$, $y = ?$

$x = 2$, $y = ?$

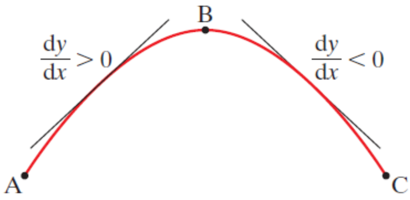
$y = (4)^3 - 9(4)^2 + 20(4) - 8 = -8$

$y = (2)^3 - 9(2)^2 + 20(2) - 8 = -4$

points are $(4, -8)$ and $(2, -4)$

Increasing and decreasing functions

Examining the given curve from left to right, we can see that it is rising (or increasing) from A to B, and decreasing from B to C.



From A to B, the slope of the tangent $\left(\text{i.e. } \frac{dy}{dx}\right)$ is positive.

From B to C, the slope of the tangent is negative.

Remember

When the curve is increasing, $\frac{dy}{dx} > 0$.

When the curve is decreasing, $\frac{dy}{dx} < 0$.

Example 3

Find the interval for which the function $f(x) = x^3 - 3x^2 - 9x + 9$ is

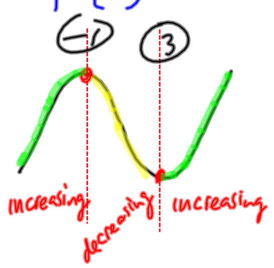
(i) increasing (ii) decreasing

Slope of curve?

$f'(x) = 3x^2 - 6x - 9$

Increasing?
$f'(x) > 0$ $\Rightarrow$ $3x^2 - 6x - 9 > 0$

decreasing
$f'(x) < 0$ $\Rightarrow$ $3x^2 - 6x - 9 < 0$



If $3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1 \; , \; x = 3$

increasing $-1 > x > 3$

decreasing $-1 < x < 3$

Example 4

Show that the curve $y = x^3 + 3x^2 + 6x$ is increasing for all values of $x \in R$.

Slope of curve?

increasing $\Rightarrow \frac{dy}{dx} > 0$

Complete square

$\frac{dy}{dx} = 3x^2 + 6x + 6$

Is $3x^2+6x+6$ positive always?

$3[x^2+2x+2]$

$3[(x+1)(x+1)]$

$3(x+1)^2 > 0$

QED