Finding the constant of integration

Each of the examples above contain an arbitrary constant c.
This arbitrary constant is generally called the **constant of integration**.
This constant of integration can be found if further information about the function is given.
This is illustrated in the following example.

Example 3

A curve with equation $y = f(x)$ passes through the point $(2, 0)$.
If $f'(x) = 3x^2 - \frac{1}{x^2}$, find $f(x)$.

input output

$$f(x) = \int \left(3x^2 - \frac{1}{x^2}\right) dx$$

$$= \frac{\cancel{3}x^3}{\cancel{3}} + \frac{x^{-1}}{+1} + c$$

$$f(x) = x^3 + \frac{1}{x} + c$$

$$f(2) = (2)^3 + \frac{1}{2} + c = 0$$

$$\Rightarrow 8\tfrac{1}{2} + c = 0 \quad \Rightarrow c = -8\tfrac{1}{2}$$

$$\Rightarrow f(x) = x^3 + \frac{1}{x} - 8\tfrac{1}{2}$$

2. Find each of these integrals:

(iv) $\int -\frac{2}{x^3}\, dx$ (v) $\int \sqrt{x}\, dx$ (vi) $\int 3x^{\frac{1}{2}}\, dx$

"Increase the power and divide by the new power!"

Rewrite in index form

Constants can be placed in front of integral

(iv) $\int -\frac{2}{x^3}\, dx = -2 \int x^{-3}\, dx$

$$= -2\left(\frac{x^{-2}}{-2}\right) + c = x^{-2} + c$$

(v) $\int \sqrt{x}\, dx = \int x^{\frac{1}{2}}\, dx$

$$= \frac{x^{3/2}}{\left(\frac{3}{2}\right)} + c = \frac{2}{3} x^{3/2} + c$$

(vi) $\int 3x^{\frac{1}{2}}\, dx = 3 \int x^{\frac{1}{2}}\, dx$

$$= 3 \frac{x^{3/2}}{\left(\frac{3}{2}\right)} + c = 2x^{3/2} + c$$

9. If $\int(6x + 5)\,dx = 19$ when $x = 2$, find the constant of integration.

$f(x) = \int (6x+5)\,dx$

$= \frac{6x^2}{2} + 5x + C$

$= 3x^2 + 5x + C$

$f(2) = 19 \Rightarrow 3(2)^2 + 5(2) + C = 19$

$12 + 10 + C = 19$

$C = -3$

$\Rightarrow f(x) = 3x^2 + 5x - 3$