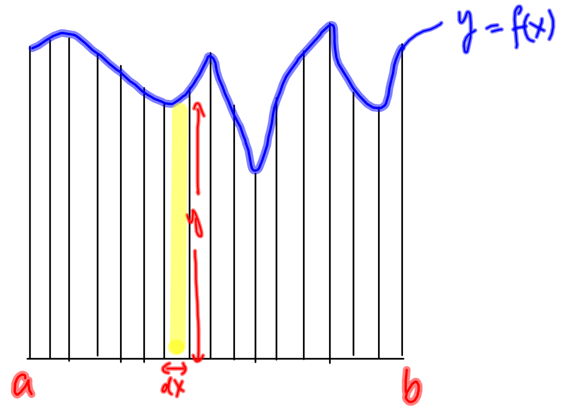
Understanding Integration

limits

$$Area = \int_a^b f(x) \cdot dx$$

sum small areas

height (y) x width

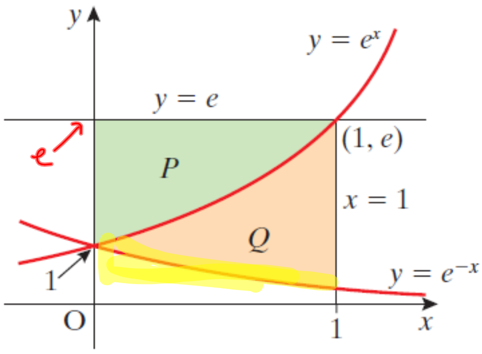


24. The given figure shows parts of the graphs of $y = e^x$ and $y = e^{-x}$.

The figure also shows two enclosed regions P and Q.

(i) Find the area of region P.

(ii) Find the area of region Q in terms of e.



Between curve and x-axis

$$\boxed{\text{Area} = \int_a^b f(x)\,dx}$$

Between curve and y-axis

$$\boxed{\text{Area} = \int_a^b x\,.dy}$$

Integration

$f'(x)$	$\to$	$f(x)$
e^x	$\to$	e^x
e^{ax}	$\to$	$\frac{1}{a}e^{ax}$

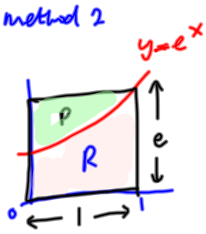
(i) consider to y axis

method 1

$y = e^x \Rightarrow x = \log_e y = \ln y$

Area $P = \int_1^e \ln y\,.dy$ can't do this!

method 2



Area Rectangle $= (1)(e) = e$

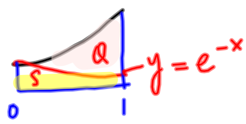
Area $R = \int_0^1 e^x.dx = [e^x + c]_0^1$

$= [e^1 + \cancel{c}] - [e^0 + \cancel{c}]$

$= e - 1$

Area $P = e - [e - 1] = 1$

(ii)



Area $S = \int_0^1 e^{-x}.dx = [-e^{-x} + c]_0^1$

$= [-e^{-1} + \cancel{c}] - [-e^{-0} + \cancel{c}]$

$= -e^{-1} + 1$

Area Q = Area R − Area S

Area $Q = (e - 1) - (-e^{-1} + 1)$

$= e - 1 + e^{-1} - 1$

$= e + e^{-1} - 2$