Differentiation 1: Tutorial Questions

Calcalas | **Calculus**

Díorthaigh | **Derivatives**

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
e^{ax}	ae^{ax}
a^x	$a^x \ln a$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\tan x$	$\sec^2 x$
$\cos^{-1}\frac{x}{a}$	$-\frac{1}{\sqrt{a^2-x^2}}$
$\sin^{-1}\frac{x}{a}$	$\frac{1}{\sqrt{a^2-x^2}}$
$\tan^{-1}\frac{x}{a}$	$\frac{a}{a^2+x^2}$

Riail an toraidh — **Product rule**

$$y = uv \quad \Rightarrow \quad \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Riail an lín — **Quotient rule**

$$y = \frac{u}{v} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Cuingriail — **Chain rule**

$$f(x) = u(v(x)) \quad \Rightarrow \quad f'(x) = \frac{du}{dv}\frac{dv}{dx}$$

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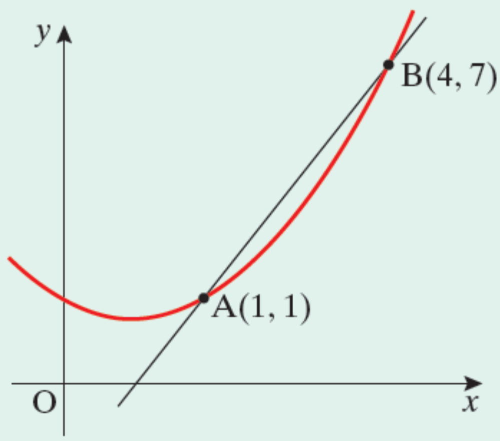
emaths.ie "Differentiation"

Link to YouTube Playlist

Example 1

Section 1: Average rate of change

Find the average rate of change of y with respect to x for the function $y = f(x)$ over the interval $[1, 4]$ as shown.



Example 2

Section 1: Average rate of change

The temperature T (°C) in a classroom on a particular day can be modelled by the equation

$$T = \frac{200}{t^2 + 2t + 20}, \text{ where } t \text{ is the time after 6.00 p.m..}$$

Find (i) the temperature in the room at 6.00 p.m.
(ii) the temperature in the room at midnight
(iii) the average rate of change of temperature from 6.00 p.m. to midnight.

Example 1

Section 2: Differentiation by 1st Principles

Differentiate $f(x) = 3x + 8$ from first principles.

Example 2

Differentiate $f(x) = x^2 - 6x$ from first principles.

Example 3

Find, from first principles, the slope of the tangent to the curve with equation $f(x) = x^2 + x + 5$ at the point where $x = 3$.

Example 1

Find the derivative of $f(x) = 6x^3 - 3x^2 + 4x$.
Hence, find $f'(2)$ and interpret the result.

Section 3: Differentiation by Rule: Powers

Example 2

Find $\frac{dy}{dx}$ for each of the following:

(i) $y = 3x^2 + 2/x$
(ii) $y = \sqrt{x} - 4/x^2$

Example 3

Find the slope of the tangent to the curve $y = 3x^2 + 4x - 5$ at the point $(1, 2)$. Hence, find the equation of the tangent at this point.

Example 4

Find the points on the curve $y = x^3 - 3x^2$ at which the slope of the tangent to the curve is 9.

Example 1

Section 4: The Product Rule

If $y = (6x^2 + 2x)(3x - 2)$, find $\frac{dx}{dy}$.

Example 2

Section 4: The Quotient Rule

If $f(x) = \frac{x^2 + 7}{3x - 1}$, find $f'(x)$.

Example 3

Find $\frac{dy}{dx}$ if (i) $y = (2x^2 - 1)^3$ (ii) $y = \sqrt{3x^2 - 2}$.

Example 4

Find $\frac{dy}{dx}$ if (i) $y = (x^2 - 3x)^4$ (ii) $y = \sqrt{x^2 - 6x}$.

Section 4: Quotient Rule with Chain Rule

Example 5

If $y = \dfrac{x}{\sqrt{1-x}}$, evaluate $\dfrac{dy}{dx}$ when $x = -3$.

Section 5: Higher derivatives

Example 1

Given that $y = x + \dfrac{1}{x}$, find $\dfrac{d^2y}{dx^2}$.

Section 5: Higher derivatives

Example 2

If $y = \frac{3}{x} + 4x$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$; hence, show that $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$.

Section 6: Trigonometric Functions

Example 1

Differentiate each of the following with respect to x:

(i) $y = 3\sin x + 2\cos x$ (ii) $y = x^2 \sin x$

Example 2

Find the derivative of each of the following:

(i) $\cos(7x - 3)$ (ii) $\tan^2 3x$ (iii) $\sin^3(x^2 + 2)$

Section 6: Trigonometric Functions

Example 3

If $f(x) = \dfrac{1 + \sin x}{\cos x}$, show that $f'(x) = \dfrac{1 + \sin x}{\cos^2 x}$ and hence evaluate $f(\pi)$.

Example 1

If $y = \sin^{-1}\frac{5x}{3}$, find $\frac{dy}{dx}$.

Section 7: Inverse Trigonometric Functions

Example 2

If $y = \tan^{-1}(2x + 1)$, find $\frac{dy}{dx}$.

Example 1

Find $\frac{dy}{dx}$ for each of the following:

(i) $y = 5e^{x^2}$ (ii) $y = e^{\cos x}$ (iii) $y = (e^x + 1)^4$

Section 8: Exponential Functions with base *e*

Example 2

If $y = e^{2x} \cos 2x$, find the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{8}$.

Example 1

Find $\dfrac{dy}{dx}$ if (i) $y = \log_e(4x^2 + 1)$ (ii) $y = \log_e(\sin^2 x)$.

Example 2

Given that $y = \log_e\left(\dfrac{1+x}{1-x}\right)$, show that $(1 - x^2)\dfrac{dy}{dx} = 2$.