

Arithmetic Sequences and Series

HW for collection on Monday 28 Jan 2013

Formulae

$$T_n = a + (n-1)d$$

$$S_n = \frac{n(a + T_n)}{2} \quad \text{or} \quad S_n = \frac{n}{2} [2a + (n-1)d]$$

$$T_n = S_n - S_{n-1}$$

a = 1st term

d = common difference

T_n = term n

S_n = sum 1st n terms

A sequence is arithmetic if

there is a common difference

between each subsequent term.

$$\text{ie. } T_n - T_{n-1} = d$$

1. Find a and d for each of the following arithmetic sequences:

(i) 2, 5, 8, ...

(ii) 7, 12, 17, ...

(iii) 0, 3, 6, 9, ...

(iv) -2, 1, 4, ...

(v) 60, 55, 50, ...

(vi) 6, 1, -4, ...

2. If $T_n = 3n - 4$, find T_1 , T_3 and T_{10} .

3. In the arithmetic sequence 2, 6, 10, 14, ... write down
 (i) a , the first term (ii) d , the common difference
 (iii) T_n and hence the value of T_{20}

4. The n th term of an arithmetic sequence is given by $T_n = 4n - 1$.
 Write down the first three terms of this sequence.
 Hence find the value of a and the value of d .

5. Find T_n of the arithmetic sequence 1, 3, 5, ...
 For what value of n is $T_n = 87$?

6. Complete the table of values for this sequence of matchstick patterns.



Number of triangles	1	2	3	4	5
Number of matchsticks	3				

- (i) Explain why the sequence generated by the numbers of matchsticks is an arithmetic sequence.
- (ii) Write an expression for the n th term of the sequence.
- (iii) How many matchsticks are needed for the 30th term of the sequence?
- (iv) Which term of the pattern has 81 matchsticks?

Grid area for question 6.

7. For the arithmetic series $2 + 5 + 8 + \dots$,

- (i) find the value of a and d
- (ii) find the sum of the first 12 terms.

Grid area for question 7.

8. Find the sum of the first 20 terms of the series

$$3 + 7 + 11 + 15 + \dots$$

9. In an arithmetic sequence, $T_4 = 14$ and $T_9 = 34$.

Find the values of a and d and hence write down the value of T_{13} .

10. In an arithmetic sequence, $T_3 = 4$ and $T_{10} = -17$.
 Find the values of a and d .
 Write down T_n of the sequence and find the value of n for which $T_n = -47$.

11. In an arithmetic sequence, $T_1 + T_3 = 12$ and $T_4 + T_6 = 24$.
 Find the values of a and d .

12. The first four terms of a series are $7 + 10 + 13 + 16 + \dots$
 Find S_8 , the sum of the first eight terms.

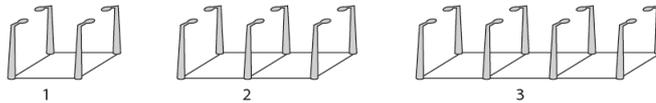
13. In an arithmetic series the n th term, $T_n = 5n - 2$.
 Find the values of a and d and hence find S_{16} of the series.

14. In an arithmetic series, $T_3 = 0$ and $T_8 = 10$.
 Find the values of a and d and hence find S_n of the series.
 How many terms of the series must be added so that their sum is 36?

15. The n th term of an arithmetic series is $T_n = 52 - 4n$.
 (i) Find the values of a and d .
 (ii) Find which term is zero.
 (iii) Find the sum of the terms which are positive.

16. In an arithmetic sequence, the sixth term is 20 and the tenth term is four times the second term.
Find the values of a and d . Hence calculate T_{100} .

17. Lamp-posts are put at the end of every 100 m stretch of a motorway, as shown,



- (i) How many lamp-posts are needed for 500 m of motorway?
- (ii) Write down, as a number sequence, the number of lamp-posts required for 100 m, 200 m, 300 m, 400 m, ...
- (iii) Find an expression in n for the n th term of this sequence.
- (iv) Use the expression found in (iii) to write down the number of lamp-posts needed for 8 km of motorway.
- (v) The M51 is a motorway being built. The contractor has ordered 2402 lamp-posts. How long is this motorway?