Question 5 **(25 marks)**

(a) $3 + 2i$ is a root of $z^2 + pz + q = 0$, where $p, q \in \mathbb{R}$, and $i^2 = -1$.
Find the value of p and the value of q.

(b) **(i)** $v = 2 - 2\sqrt{3}i$. Write v in the form $r(\cos\theta + i\sin\theta)$,
where $r \in \mathbb{R}$ and $0 \leq \theta \leq 2\pi$.

(ii) Use your answer to **part (b)(i)** to find the **two** possible values of w, where $w^2 = v$.
Give your answers in the form $a + ib$, where $a, b \in \mathbb{R}$.

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Q5	Model Solution – 25 Marks	Marking Notes
(a)	$3-2i =$ other root $-p=(3+2i)+(3-2i)=6$ $p=-6$ $q=(3+2i)(3-2i)=13$ Or $(3+2i)^2+p(3+2i)+q=0$ $5+12i+3p+2pi+q=0$ $2p=-12 \Rightarrow p=-6$ $5+3p+q=0 \Rightarrow q=13$ Or $\frac{-p\pm\sqrt{p^2-4q}}{2}=3\pm 2i$ $-p\pm\sqrt{p^2-4q}=6\pm 4i$ $-p=6$ $\therefore p=-6$ $\sqrt{4q-p^2}=4$ $4q-p^2=16$ $4q-(-6)^2=16$ $4q=52$ $\therefore q=13$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - Second root identified *High Partial Credit:* - Sum and product of roots formulated into equations for p and q - p or q found correctly *Low Partial Credit:* - Root substituted into equation - Any correct substitution *High Partial Credit:* - Real and imaginary terms formulated into equations for p and for q *Low Partial Credit:* - Some substitution into quadratic formula *High Partial Credit:* - Finds p - Full substitution into quadratic formula and equated to either root.

(b) **(i)**	$\|v\| = \sqrt{4+12} = 4$ $\theta = 300°$ $v = 4(\cos 300° + i \sin 300°)$ Or $\|v\| = \sqrt{4+12} = 4$ $\theta = \frac{5\pi}{3}$ $v = 4\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - Correct plot on the Argand diagram - Some use of Pythagoras to find modulus - Some use of trigonometry to find argument *High Partial Credit:* Modulus or argument found *Note*: Accept $4\left(\cos -\frac{\pi}{3} + i\sin -\frac{\pi}{3}\right)$ and $4(\cos -60° + i\sin -60°)$
(b) **(ii)**	$w = \pm v^{\frac{1}{2}}$ $w = \pm 2(\cos 300 + i\sin 300)^{\frac{1}{2}}$ $w = \pm 2(\cos 150 + i\sin 150)$ $w = \pm(-\sqrt{3} + i)$ $w = -\sqrt{3} + i$ or $\sqrt{3} - i$ Or $w = [4(\cos(300 + 360n) + i\sin(300 + 360n)]^{\left(\frac{1}{2}\right)}$ $w = 4^{\frac{1}{2}}[\cos(150 + 180n) + i\sin(150 + 180n)]$ $\underline{n = 0}$ $w = 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -\sqrt{3} + i$ $\underline{n = 1}$ $w = 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \sqrt{3} - i$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - w written in polar form with index - Some use of De Moivre's Theorem - $w = v^{\frac{1}{2}}$ *High Partial Credit:* - De Moivre's theorem applied to *w* - One solution found - Solutions in polar form *Note*: Accept candidates answer from (b)(i)