(a) Let $u = \dfrac{1+3i}{3+i}$ where $i^2 = -1$.

(i) Express u is the form $a + ib$ where $a, b \in \mathbf{R}$.

(ii) Evaluate $|u|$.

(i) $u = \dfrac{(1+3i)(3-i)}{(3+i)(3-i)} = \dfrac{3 - 1i + 9i \pm 3\cancel{i^2}}{9 \pm 1\cancel{i^2}} = \dfrac{6+8i}{10} = \dfrac{3}{5} + \dfrac{4}{5}i$

(ii) $|u| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25} + \frac{16}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1} = 1$

(ii) $w_1 = a + ib$ and $w_2 = c + id$. Prove that $\overline{(w_1 w_2)} = (\overline{w_1})(\overline{w_2})$, where $\overline{w}$ is the complex conjugate w.

$w_1 w_2 = ac + adi + cbi \mp bd\cancel{i^2}$

$= ac - bd + adi + cbi$

$\overline{w_1 w_2} = ac - bd - adi - cbi$

$\overline{w_1} = a - bi$

$\overline{w_2} = c - di$

$\overline{w_1}\,\overline{w_2} = ac - adi - cbi \mp bd\cancel{i^2}$

$= ac - bd - adi - cbi$

QED