Q.1 Express $2x^2 - 4x - 5$ in the form $a(x+h)^2 + k$ and hence,

(i) solve the equation $2x^2 - 4x - 5 = 0$

(ii) find the minimum point of this curve.

$$2x^2 - 4x - 5 = 2\left[x^2 - 2x - 5/2\right] = 0$$

	x	-1
x	x^2	$-x$
-1	$-x$	$+1$

$$= 2\left[(x^2 - 2x + 2 - 1 - \tfrac{5}{2}\right] = 0$$

$$= 2\left[(x-1)^2 - \tfrac{7}{2}\right] = 0$$

$$= 2(x-1)^2 - 7 = 0$$

(i) hence solve :

$$2(x-1)^2 = 7$$

$$(x-1)^2 = 7/2$$

$$(x-1) = \pm\sqrt{7/2}$$

$$x = 1 \pm \sqrt{\tfrac{7}{2}}$$

(ii) min. pt. $= (1, -7)$

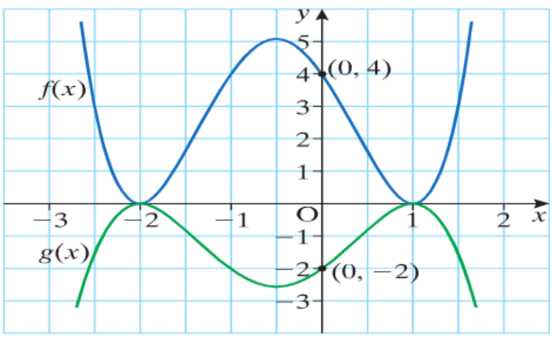
Q.2

The graphs of two functions $f(x)$ and $g(x)$ are given in the following diagram.

If $f(x) = ag(x)$,

(i) find the value of a

(ii) find equations for $f(x)$ and $g(x)$.



(i) $f(x) = a\,g(x)$

$$f(0) = a\,g(0)$$

$$4 = -2a$$

$$a = -\frac{4}{2} = -2$$

(ii) $f(x)$ is a polynomial of degree 4

$$f(x) = n(x+2)(x+2)(x-1)(x-1)$$

$$f(0) - n(0+2)(0+2)(0-1)(0-1) = 4$$

$$4n = 4 \Rightarrow n = 1$$

$$f(x) = (x+2)(x+2)(x-1)(x-1)$$

$$g(x) = -2(x+2)(x+2)(x-1)(x-1)$$

Q.3 Show that the roots of the equation $x^2 - (a + d)x + (ad - b^2) = 0$ are real.

If Roots are real $\Delta \geq 0$ $\Delta = b^2 - 4ac$

$$\Delta = (-(a+d))^2 - 4(1)(ad - b^2)$$

$$= a^2 + 2ad + d^2 - 4ad + 4b^2$$

$$= a^2 - 2ad + d^2 + 4b^2$$

$$= (a-d)^2 + 4b^2 \geq 0$$

Q.4 Given that $(x + 1)$ and $(x - 2)$ are factors of $6x^4 - x^3 + ax^2 - 6x + b$, find the values of a and b.

$$f(-1) = 6(-1)^4 - (-1)^3 + a(-1)^2 - 6(-1) + b = 0$$

$$6 + 1 + a + 6 + b = 0$$

$$a + b = -13$$

$$f(2) = 6(2)^4 - (2)^3 + a(2)^2 - 6(2) + b = 0$$

$$96 - 8 + 4a - 12 + b = 0$$

$$4a + b = -76$$

SOLVE

$$\begin{array}{rl} 4a + b &= -76 \\ -a - b &= 13 \\ \hline 3a &= -63 \end{array}$$

$$a = -21$$

$$a + b = -13$$

$$-21 + b = -13$$

$$b = 8$$

Q.5

A cubic function f is defined for $x \in \mathbb{R}$ as

$$f : x \mapsto x^3 + (1 - k^2)x + k, \quad \text{where } k \text{ is a constant.}$$

(a) Show that $-k$ is a root of f. ie.. show $f(-k) = 0$

$f(k) = (-k)^3 + (1-k^2)(-k) + k$

$= -k^3 - k + k^3 + k = 0$ QED

(b) Find, in terms of k, the other two roots of f.

DIVIDE By FACTOR

$$\begin{array}{r l}
 & x^2 - kx + 1 \\
x + k \,\big) & \cancel{x^3} + 0x^2 + (1-k^2)x + k \\
 & \mp \cancel{x^3} \mp kx^2 \\
 & -k\cancel{x^2} + (1-k^2)x \\
 & \pm \cancel{k}x^2 \pm k^2 x \\
 & \cancel{x} + \cancel{k} \\
 & \mp \cancel{x} \mp \cancel{k}
\end{array}$$

solve $x^2 - kx + 1 = 0$

$a = 1 \quad b = -k \quad c = 1$

$$x = \frac{+k \pm \sqrt{k^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{k \pm \sqrt{k^2 - 4}}{2}$$