Algebra Revision Worksheet

Leaving Cert. Higher Level

Project Maths

FACTORS & FRACTIONS

Factorise:

i $3p^2 + 6pq$

ii $6ab + 12bc - 8ac - 96^2$

iii $3x^2 - 12y^2$

iv $2x^2 - 7x - 15$

v $64 - 27x^3$

vi $4a^3 + 32b^3$

Simplify : $\dfrac{x^3+7x^2+12x}{x^2+2x-3}$

Simplify. : $\dfrac{5}{2x-3} - \dfrac{3}{2x^2-3x} - \dfrac{1}{x}$

Simplify : $\dfrac{a+b}{\frac{1}{a}+\frac{1}{b}}$

$f(x) = \dfrac{x^3-1}{x^2-1}$, $x \neq 1$

$g(x) = \dfrac{x^2+x+1}{x^2-x-2}$, $x \neq -1, 2$

If $f(x) \div g(x) = ax+b$, find the value of a and b.

MANIPULATING FORMULAE

If $c = \frac{b^2 - ac}{b + a}$, express a in terms of the other variables.

If $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, express v in terms of the other variables. If $f = 15$ and $u = 20$ find the value of v?

$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$ express r in terms of the other variables

POLYNOMIAL IDENTITIES

$x^2 - 6x + t = (x+k)^2$ where t and k are constants. Find the value of k and t.

$ay^2 + by(y-4) + c(y-4) = y^2 + 13y - 20$
find the value of constants a, b and c.

SURDS

Express in its simplest Surd form :

i) $\sqrt{99}$

ii) $\frac{1}{2}\sqrt{112}$

iii) $\sqrt{2\frac{1}{4}}$

iv) $\sqrt{180} + \sqrt{20} - \sqrt{125}$

Express $\frac{1}{3\sqrt{5}} - \frac{1}{2\sqrt{20}}$ in the form $k\sqrt{5}$

Express $\frac{-4}{\sqrt{5}-3}$ in the form $\sqrt{5}+a$

Express $\frac{1-\sqrt{3}}{1+\sqrt{3}}$ in the form $a\sqrt{3}-b$

QUADRATIC EQUATIONS & FUNCTIONS

Solve $3x^2-5x-12=0$

Solve : $2x^2 - 7x + 4 = 0$, leave your answer in surd form

form a quadratic equation with roots $\frac{1}{2}$ and 3

Verify $2x^2 - x + 3 = 0$ has no Real Roots

Solve $\frac{1}{x+1} + \frac{4}{2x-1} = \frac{5}{3}$

MODULUS EQUATIONS

Solve $|3x-2|=4$

Solve $2|x+1|-|x+3|=0$

SURD EQUATION

Solve $x = \sqrt{3x+7} - 1$

COMPLETE SQUARE (VERTEX FORM) OF A QUADRATIC EQUATION

Express $x^2 - 6x - 12 = 0$ in the form $(x-p)^2 + q = 0$.
Hence solve for x.

	x	-3
x	x^2	$-3x$
-3	$-3x$	$+9$

$$x^2 - 6x + 9 - 9 - 12 = 0$$
$$(x-3)^2 - 21 = 0$$

Hence Solve

$$(x-3)^2 = 21$$
$$x - 3 = \pm\sqrt{21}$$
$$x = 3 \pm \sqrt{21}$$

Express $-x^2 + 11x + 15 = 0$ in the form $q - (x-p)^2 = 0$ and hence solve.

$$-\left[x^2 - 11x - 15\right] = 0$$

$$-\left[\left(x - \tfrac{11}{2}\right)^2 - \tfrac{121}{4} - 15\right] = 0$$

$$-\left[\left(x - \tfrac{11}{2}\right)^2 - \tfrac{181}{4}\right] = 0$$

$$\tfrac{181}{4} - \left(x - \tfrac{11}{2}\right)^2 = 0$$

Hence:

$$\tfrac{181}{4} = \left(x - \tfrac{11}{2}\right)^2$$

$$\pm\sqrt{\tfrac{181}{4}} = x - \tfrac{11}{2}$$

$$x = \tfrac{11}{2} \pm \sqrt{\tfrac{181}{4}}$$

	x	$-\frac{11}{2}$
x	x^2	$-\frac{11}{2}x$
$-\frac{11}{2}$	$-\frac{11}{2}x$	$+\frac{121}{4}$

CUBIC EQUATIONS & FUNCTIONS

$f(x) = 3x^3 + mx^2 - 17x + n$ where n and m are constants. Given that $x-3$ and $x+2$ are factors of $f(x)$. find the value of m and n.

Solve $4x^3 + 10x^2 - 7x - 3 = 0$

$x^2 - t$ is a factor of $x^3 - px^2 - qx + r$

i) Show that $pq = r$

ii) Express the roots of $x^3 - px^2 - qx + r = 0$ in terms of p and q.

Inequalities

Solve $11 - 3x \geq 2$, $x \in \mathbb{N}$ and show the solution set on a number line.

i) Solve Set A — Set A : $2x + 7 \leq 11$, $x \in R$

ii) Solve Set B — Set B : $4 - 2x < 10$, $x \in R$

iii) Find $A \cap B$ and show on a no. line.

Solve $15 + 2x \geq x^2$, $x \in \mathbb{R}$

Solve $12 < x^2 + 4x$, $x \in \mathbb{R}$

Solve $\frac{3x+1}{x-1} \geq 2$, $x \in \mathbb{R}$, $x \neq 1$

$f(x) = x^2 - 7x + 12$

i) Show if $f(x+1) \neq 0$ then

$$\frac{f(x)}{f(x+1)} = \frac{x-4}{x-2}$$

ii) find the Range of values of x for which

$$\frac{f(x)}{f(x+1)} > 3$$

Solve $|5-2x| \leq 3$

Solve $|4x+7| > 1$

$f(x) = |x+1|$ and $g(x) = 4$

i) Solve $f(x) = g(x)$

ii) Solve $f(x) \leq g(x)$

Prove for all $a, b \in \mathbb{R}$ that:

$a^2 + 4b^2 \geq 4ab$

Show $\frac{a}{b^2} + \frac{b}{a^2} \geq \frac{1}{a} + \frac{1}{b}$

If $a+b \geq 0$ and $a, b \neq 0$

SIMULTANEOUS EQUATIONS

Solve $\frac{2x-5}{3} + \frac{y}{5} = 6$

$\frac{3x}{10} + 2 = \frac{3y-5}{2}$

Solve

$$3x + y + z = 0$$
$$x - y + z = 2$$
$$2x - 3y - z = 9$$

Craig invested €50,000 in three funds paying 6%, 8% and 10% interest p.a. The total earned in year 1 was €3700. Twice as much was invested at 6% as was 10%. How much was invested in each fund?

Solve : $2x + y = 3$
$x^2 + xy + y^2 = 3$

Indices, Index Equations

Simplify

i) $125^{2/3}$

ii) $32^{2/5} - 81^{1/4}$

iii) $\frac{4^{-\frac{1}{2}}}{64^{2/3}}$

Solve: $27^{4+3x} = 243^{1+2x}$

Solve $2^{x^2} = 8^{2x+9}$

Solve

$2^x = 8^{y+1}$

$3^{x-9} = 9^y$

Solve

$2^{2x+1} - 5(2^x) + 2 = 0$

Solve : $3^{x+2} - 82 + 3^{2-x} = 0$

Logs and logarithmic equations

Evaluate

i) $\log_7 343$

ii) $\log_{27} \frac{1}{3}$

Solve $\log_2(x+6) - \log_2(x+2) = 1$

Solve: $\ln(x+1) + \ln(x-1) = \ln 3$

Solve $\log_2 x - \log_2(x-1) = 4\log_4 2$

Given $p = \log_c x$

express $\log_c \sqrt{x} + \log_c cx$

in terms of p.

Loudness in dB is given by the formula $L = 10 \log I$ where I is the intensity.

i) a whisper has the intensity of 100 what is the loudness in dB.

ii) What is the Insitity of a sound that is 50 dB.

The population of Snow leopards is 6000. If no conservation is put in place the population will decrease by 11% per year. Find to the nearest year how long till the population is halved.

Solve $\log_3 x + \log_3 y = 2$

$\log_3 (2y-3) - 2\log_9 x = 1$

EXPONENTIAL FUNCTION

In an experiment the following was recorded

t	0	1	2	3
$Q(t)$	2·920	2·642	2·391	2·163

If $Q(t) = Ae^{-bt}$

i) Find the value of A and b.

ii) Find the value of constant k for which $Q(t+k) = \frac{1}{2} Q(t)$, $t \geq 0$