

1. Equation of circle containing three given points

Circle: S **Example 1**Find the equation of the circle which contains the points $A(2, 1)$, $B(0, 5)$ and $C(-1, 2)$.General Equation: $x^2 + y^2 + 2gx + 2fy + c = 0$

$$A(2, 1) \in S \Rightarrow (2)^2 + (1)^2 + 2g(2) + 2f(1) + c = 0$$

$$5 + 4g + 2f + c = 0$$

$$4g + 2f + c = -5 \quad (1)$$

$$B(0, 5) \in S \Rightarrow (0)^2 + (5)^2 + 2g(0) + 2f(5) + c = 0$$

$$25 + 10f + c = 0$$

$$10f + c = -25 \quad (2)$$

$$C(-1, 2) \in S \Rightarrow (-1)^2 + (2)^2 + 2g(-1) + 2f(2) + c = 0$$

$$5 - 2g + 4f + c = 0$$

$$-2g + 4f + c = -5 \quad (3)$$

$$\begin{array}{r} 4g + 2f + c = -5 \quad (1) \\ -2g + 4f + c = -5 \quad (3) \\ \hline \end{array} \Rightarrow \begin{array}{r} 4g + 2f + c = -5 \\ -4g + 8f + 2c = -10 \\ \hline \end{array}$$

$$(4) \quad 10f + 3c = -15$$

$$\begin{array}{r} 10f + c = -25 \quad (2) \\ -10f - 3c = +15 \quad (4) \\ \hline \end{array}$$

$$-2c = -10 \Rightarrow c = 5$$

$$\text{Sub into (2)} \Rightarrow 10f + 5 = -25$$

$$10f = -30 \Rightarrow f = -3$$

$$\text{Sub into (1)} \Rightarrow 4g + 2(-3) + 5 = -5$$

$$4g - 6 + 5 = -5$$

$$4g = -4 \Rightarrow g = -1$$

$$\text{Equation: } x^2 + y^2 - 2x - 6y + 5 = 0$$