4. Determine whether the circles $\underbrace{x^2 + y^2 - 16y + 32 = 0}_{S_1}$ and $\underbrace{x^2 + y^2 - 18x + 2y + 32 = 0}_{S_2}$ touch internally or externally.

S_1 Centre $C_1 = (0, 8)$

$R = \sqrt{g^2 + f^2 - c}$

$R_1 = \sqrt{0^2 + 8^2 - 32} = 4\sqrt{2}$

S_2 Centre $C_2 = (9, -1)$

Radius $R_2 = \sqrt{9^2 + 1^2 - 32} = 5\sqrt{2}$

distance between the centres

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(9-0)^2 + (-1-8)^2} = \sqrt{81 + 81}$

$d = 9\sqrt{2}$

$R_1 + R_2 = 4\sqrt{2} + 5\sqrt{2} = 9\sqrt{2} = d$

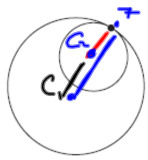
$\Rightarrow$ externally

5. $\underbrace{x^2 + y^2 - 4x - 6y + 5 = 0}_{S_1}$ and $\underbrace{x^2 + y^2 - 6x - 8y + 23 = 0}_{S_2}$ are two circles.

(i) Prove that the circles touch internally.

(ii) Find the equation of the common tangent.

(iii) Hence find the coordinates of the point of contact of the two circles.



$C_1 = (2, 3)$ $R_1 = \sqrt{2^2 + 3^2 - 5} = 2\sqrt{2}$

$C_2 = (3, 4)$ $R_2 = \sqrt{3^2 + 4^2 - 23} = \sqrt{2}$

$d = \sqrt{(3-2)^2 + (4-3)^2} = \sqrt{2}$

Is $R_1 - R_2 = d$?

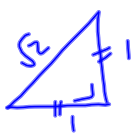
$R_1 - R_2 = 2\sqrt{2} - \sqrt{2} = \sqrt{2} = d$

$\Rightarrow$ internally touch.

$S_1 - S_2 =$ common tangent or chord

$$\begin{array}{r} \cancel{x^2} + \cancel{y^2} - 4x - 6y + 5 = 0 \\ - \cancel{x^2} - \cancel{y^2} + 6x + 8y - 23 = 0 \\ \hline 2x + 2y - 18 = 0 \end{array}$$

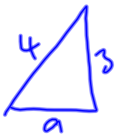
Tangent: $x + y - 9 = 0$

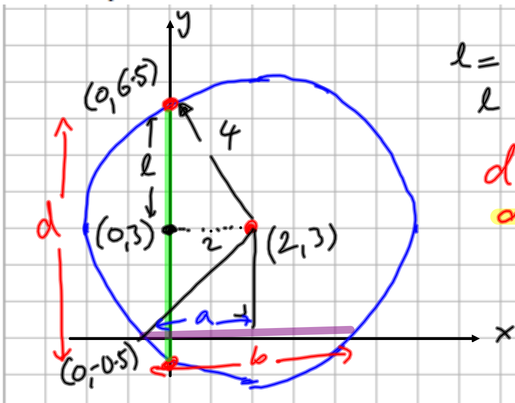


$C_1 \xrightarrow{\sqrt{2}} C_2 \xrightarrow{\sqrt{2}} T$

$(2,3) \rightarrow (3,4) \rightarrow (4, 5)$

7. A circle has centre (2, 3) and radius 4 units in length.
   (i) Draw a sketch of this circle.
   (ii) Show that the distance between the points where the circle intersects the y-axis is $4\sqrt{3}$.
   (iii) Find the length of the intercept the circle cuts off the x-axis. $2\sqrt{7}$





$\ell = \sqrt{4^2 - 2^2}$

$\ell = 2\sqrt{3}$

$d = 2\ell$

$d = 2(2\sqrt{3})$

$d = 4\sqrt{3}$

$a = \sqrt{4^2 - 3^2} = \sqrt{7}$

$b = 2\sqrt{7}$