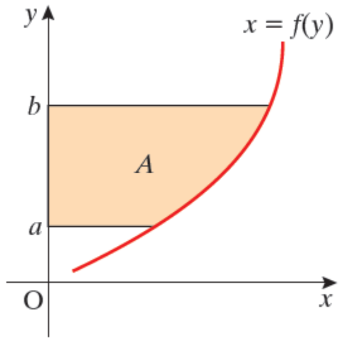
Area between a curve and the *y*-axis

If we require the area between a curve and the y-axis, the function must be written in the form $\boldsymbol{x = f(y)}$.

The area of the shaded region between the curve and the y-axis between the lines $y = b$ and $y = a$ is given by:

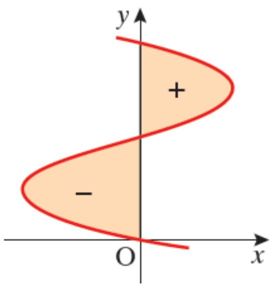
Area between a curve and the y-axis

$$\text{Area } A = \int_a^b x\,dy$$

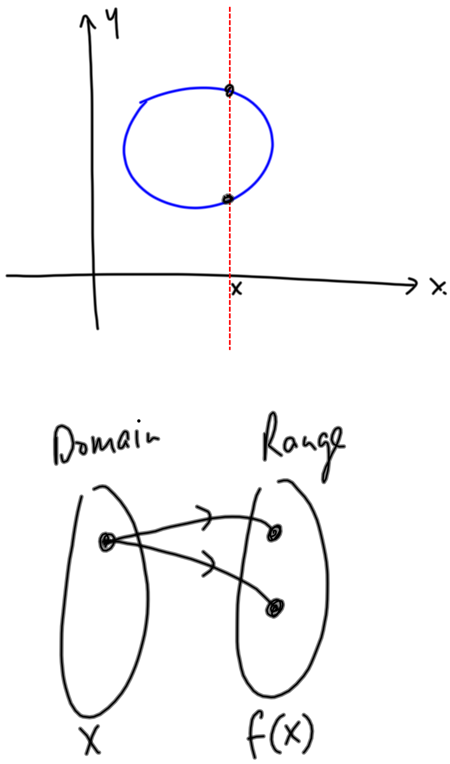


If the region is to the right of the y-axis, the area is positive; if the region is to the left of the y-axis, the area is negative.

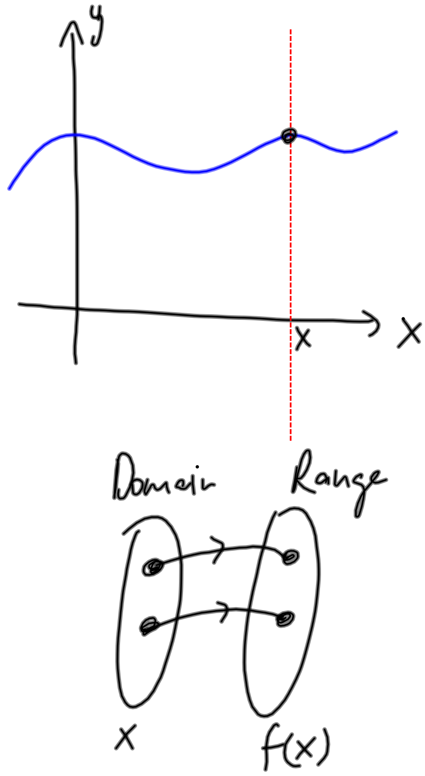
Areas to the right and to the left of the y-axis must be found separately and then added.



Not a function



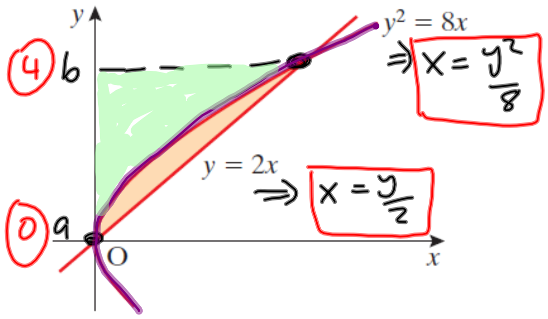
Is a function



In a function each input can only have one output.

16. The diagram shows the curve $y^2 = 8x$ and the line $y = 2x$.

Find the points of intersection of the line and the curve and hence find the area enclosed by the line and curve.



Area between a curve and the y-axis

Area $A = \int_a^b x \, dy$

PLAN

$$A = \int_a^b (\text{line} - \text{curve})\, dy$$

$y = 2x$

Pts $(0,0)$

$(4,2)$

INTERSECTION?

$$\frac{y}{2} = \frac{y^2}{8}$$

$$4y = y^2$$

$$y^2 - 4y = 0$$

$$y(y-4) = 0$$

$$\boxed{y = 0, \quad y = 4}$$

LIMITS

INTEGRATE

$$A = \int_0^4 \left[\frac{y}{2} - \frac{y^2}{8}\right] dy$$

$$= \left[\frac{y^2}{4} - \frac{y^3}{24} + c\right]_0^4$$

$$= \left[\frac{(4)^2}{4} - \frac{(4)^3}{24} + \cancel{c}\right] - \left[\frac{(0)^2}{4} - \frac{(0)^3}{24} + \cancel{c}\right]$$

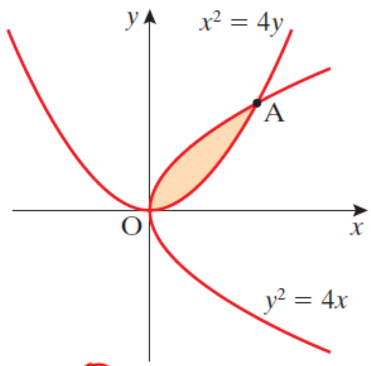
$$= 4 - \frac{8}{3} = \frac{4}{3} \text{ units}^2$$

17. The sketch on the right shows the curves

$y^2 = 4x$ and $x^2 = 4y$.

(i) Find the coordinates of the point A.

(ii) Find the area of the shaded region enclosed by the two curves.



PLAN ① INTERSECTION ?

SOLVE EQUATIONS

Rewrite ①

(*only considering positive values)

→ ②

$y^2 = 4x$ ①, $x^2 = 4y$ ②

$y = \sqrt{4x}\,^{*} \Rightarrow y = 2\sqrt{x}$

$x^2 = 4(2\sqrt{x})$

$x^2 = 8\sqrt{x} \Rightarrow x^4 = 64x$

$x^4 - 64x = 0 \Rightarrow x(x^3 - 64) = 0$

$\Rightarrow x = 0, \quad x^3 = 64$

$x = 4$

Sub back into ②

$\Rightarrow (0)^2 = 4y \Rightarrow y = 0 \Rightarrow O \overset{a}{(0,0)}$

$\Rightarrow (4)^2 = 4y \Rightarrow y = 4 \Rightarrow A \overset{b}{(4,4)}$

② INTEGRATE DIFFERENCE ?

$A = \int_a^b [g(x) - f(x)]\,dx$

Top fn : $y = 2\sqrt{x}$ ②

Bottom fn : $y = \frac{x^2}{4}$ ①

$A = \int_0^4 \left[2\sqrt{x} - \frac{x^2}{4}\right] dx$

$= \int_0^4 \left[2x^{1/2} - \frac{x^2}{4}\right] dx$

$= \left[\frac{2x^{3/2}}{\left(\frac{3}{2}\right)} - \frac{x^3}{3(4)} + c\right]_0^4$

$= \left[\frac{4}{3}x^{3/2} - \frac{x^3}{12} + c\right]_0^4$

$= \left[\frac{4}{3}(4)^{3/2} - \frac{(4)^3}{12} + \cancel{c}\right] - \left[\frac{4}{3}(0)^{3/2} - \frac{(0)^3}{12} + \cancel{c}\right]$

$A = \frac{16}{3}$ units^2